Mathematisches Institut
Heinrich-Heine-Universität Düsseldorf
Prof. Dr. Stefan Schröer

WS 21/22

Algebraic Geometry I

Sheet 5

Excercise 1. Let X be a scheme and $Z \subset X$ be a closed subscheme. Verify that the inclusion morphism $i : Z \to X$ is a *monomorphism* in the category $\mathcal{C} = (\mathrm{Sch})$ of schemes. In other words, for each scheme T and each pair of morphisms $f, g : T \to Z$ with $i \circ f = i \circ g$ we already have $f = g$.

Exercise 2. Let X be a scheme, and $X_{\mathrm{red}} \subset X$ be its reduction. Show that for each reduced scheme T, the canonical map

$$\mathrm{Hom}(T, X_{\mathrm{red}}) \longrightarrow \mathrm{Hom}(T, X)$$

is bijective.

Exercise 3. Let X be a noetherian scheme. Verify that every ascending chain $\mathscr{I}_0 \subset \mathscr{I}_1 \subset \ldots$ of quasicoherent sheaves of ideals is stationary. Conclude that every descending chain $Z_0 \supset Z_1 \supset \ldots$ of closed subschemes is stationary.

Exercise 4. Let X be a scheme and $A, B \subset X$ be two closed subschemes. Prove that among the closed subschemes $Z \subset X$ contained in both A and B there is a largest one, which is then written as $Z = A \cap B$.

Abgabe: Bis Donnerstag, den 25. November um 23:55 Uhr über ILIAS.