Abstracts of Minicourses

Steffen Kionke:

“An introduction to arithmetic groups”

Arithmetic groups are (roughly speaking) matrix groups with integral entries. They are of interest in various fields such as number theory, group theory and geometry. We will give an introduction to arithmetic groups, look at examples, discuss elementary properties and present some classical results. In particular, we will take a look at the finite quotients of arithmetic groups and discuss the congruence subgroup property. A famous conjecture of Serre characterises the congruence subgroup property as a phenomenon of “higher rank”. This conjecture is still open in some cases. At the end, we will see how the congruence subgroup property can be used to find non-isomorphic arithmetic groups with isomorphic profinite completions.

Roman Sauer:

“Higher Kazhdan property and high-dimensional expanders”

In this lecture series we will discuss cohomological and geometric aspects of higher-dimensional phenomena of the Kazhdan property. The cohomological aspect concerns vanishing theorems for group cohomology with unitary coefficients that generalize Delorme’s characterization of property (T) via the vanishing of the first cohomology. The geometric aspect concerns higher-dimensional generalizations of expander graphs in the simplicial and Riemannian context.

Here is a list of concrete topics:

- higher Kazhdan property T defined by cohomology and an analog of Garland’s theorem for simple Lie groups
- introduction to topological expander and their Riemannian counterpart – uniform waist inequalities
- construction of topological expanders from coboundary expansion – the general method
- a waist inequality in codimension 2 for Riemannian manifolds with Kazhdan fundamental group
- a conjecture connecting higher Kazhdan property (T) and waist inequalities.

Stefan Witzel:

“Euclidean buildings then and now”

An important tool in studying an arithmetic subgroup of a semisimple Lie group is the associated symmetric space. The study of S-arithmetic groups leads to considering semisimple groups over non-Archimedean fields; and the geometric counterpart to the symmetric space is a Euclidean building. These Bruhat–Tits buildings are the main reason to learn about Euclidean buildings and I will give a glimpse of the theory.

There are Euclidean buildings that do not come from semisimple groups and I will indicate why these can be very interesting as well.

Abstracts of Research Talks

Caterina Campagnolo:

"Bounded cohomology of groups: what it can do for you"

Bounded cohomology is a theory introduced in the 70s by Johnson in the context of Banach algebras, and developed by Gromov in the 80s. Since then it has become a powerful tool to study various properties of groups and spaces: for example it allows to characterize amenable groups and hyperbolic groups. At first glance a minor modification of standard cohomology, it carries interesting additional information, but is also very difficult to compute in many cases. In this talk I will present the main properties of the theory and speak of recent developments of the last years.

Pavel Gvozdevsky:

"Width of words in linear groups"

A word w is an element in a free group. Given a word w and a group G, we have the word map from G^n to G defined by substitution, where n is the rank of the free group. The set of values $w(G)$ consists of the image of this map and the inverses of elements of the image. The width of the word w in the group G is the minimal constant C such that every element of $\langle w(G) \rangle$ can be expressed as the product of C elements of $w(G)$.

The talk will be devoted to known results about width of words in certain linear groups, such as algebraic groups over an algebraically closed field, compact Lie groups, finite simple groups, general linear groups over a skew field, and Chevalley groups over commutative rings. A recent result by the speaker about the width of words in Chevalley groups over arithmetic rings will be discussed in detail.

Kristian Holm:

"Volume Formulas"

I will talk about lattices and mention various formulas that give explicit expressions for the volumes of quotients of Lie groups by different kinds of arithmetic subgroups. In particular, I will say a few things about how Prasad's Volume Formula can be used to determine lattices of minimal covolume in the symplectic group, showcasing the results of a recent joint project with Amir Džambić and Ralf Köhl.

Waltraud Lederle:

"Boomerang subgroups in arithmetic lattices"

The Nevo–Stuck–Zimmer theorem states that every ergodic probability measure preserving action of a lattice in a high rank simple Lie group is essentially transitive or essentially free. We introduce a new notion called "boomerang subgroups", and almost every point stabilizer of a p.m.p. action of a countable group on a standard probability space is a boomerang. For many arithmetic lattices, we generalize the Nevo–Stuck–Zimmer theorem to boomerang subgroups. This is joint work with Yair Glasner.

– please turn over –

Benoit Loisel:

"Action of an S-arithmetic subgroup of a Chevalley group on a building"

Let C be a smooth, geometrically integral projective curve over a field $\mathbb{F}$. If S is a finite set of closed points, we can consider the ring of integers of regular functions on C outside of S, denoted by $\mathcal{O}_S$. A goal of the theory of S-arithmetic groups is to understand the structure and properties of the groups $\mathrm{G}(\mathcal{O}_S)$ for a group scheme G.

In this talk, by adapting techniques that Mason used on the tree of SL_2, we will see that there remains, in the space of orbits for the action of an $S = \{P\}$-arithmetic subgroup of a split reductive group on its associated Bruhat–Tits building, a number of sectors of the building in connection with the Picard group of the ring $\mathcal{O}_S$. This allows us to recover some results due to Serre, or to Soulé. We will mention some perspectives in (co)homology of groups opened by the study of this action.

Chen Meiri:

"On the model theory of higher rank arithmetic groups"

In this talk we will explain the definitions of interpretability and bi-interpretability and will formalize some conjectures regarding the bi-interpretability of higher rank arithmetic groups with the ring $\mathbb{Z}$.

Eduard Schesler:

"The Sigma-invariants of S-arithmetic subgroups of Borel groups"

If one is interested in a group G, then typically among the first properties one wants to know about G is whether G is finitely generated or even finitely presented. In 1965 it was shown by C. T. C. Wall that these two properties are just the first two of an infinite sequence $(\mathsf{F}_n)_n$ of so-called finiteness properties F_n. By definition, a group G is of type F_n if there is an Eilenberg-MacLane space $K(G;1)$ with finite n-skeleton. These finiteness properties were intensively studied in the class of S-arithmetic groups and are still subject of current research. In this talk we will determine the finiteness properties of a large class of solvable S-arithmetic groups in characteristic 0. In particular we will determine the finiteness properties of every subgroup H of the group of upper triangular matrices $B_n(\mathbb{Z}[1/p]) < \mathrm{SL}_n(\mathbb{Z}[1/p])$ that contains the subgroup $U_n(\mathbb{Z}[1/p]) < B_n(\mathbb{Z}[1/p])$ of unipotent matrices, where p is any sufficiently large prime number. To do so, we will combine methods form the theory of buildings, Sigma-invariants, and discrete Morse theory.

– 20 August 2024 –