Singularities, Monodromy and Zeta Functions
Blatt 9

Exercises for discussion in the class on 21.12.2018

Recall that, for A a filtered K-algebra and M a filtered A-module with filtration $\{F_r(M)\}_r$, we say

$$M \text{ is of type } (d,e) :\Longleftrightarrow \dim_K F_r(M) = e\frac{r^d}{d!} + o(r^d).$$

Aufgabe 1:

Find the type (d,e) of the following filtered modules:

(a) The D-module M_f with respect to the good filtration discussed in the lecture;

(b) For $\underline{x}$ of length n and $k \leq n$, the $K[\underline{x}]$-module $K[\underline{x}]/(x_1, ..., x_k)$ (first consider, what is the natural filtration?);

(c) The $K[x_1, x_2]$-module $K[x_1, x_2]/(x_1 x_2)$;

(d) The $K[x, y]$-module $K[x, y]/(y^2 - x^3)$.

Aufgabe 2:

Suppose that M is a filtered A-algebra, where A is a filtered K-algebra and consider the associated grading $\mathcal{G}\mathrm{r}(M)$ as described in the lecture.

(a) Verify that $\mathcal{G}\mathrm{r}(M)$ is indeed a $\mathcal{G}\mathrm{r}(A)$-module.

(b) Prove that if $\mathcal{G}\mathrm{r}(M)$ is finitely generated as a $\mathcal{G}\mathrm{r}(A)$-module, then M is finitely generated as an A-module.

Aufgabe 3:

Are there alternative filtrations of $K[x]$ making it of type $(2, 1)$ or of type $(1, 2)$ as a $K[x]$-module?

Aufgabe 4:

If M_i are filtered A-modules of type (d_i, e_i) for $i = 1, 2$ respectively, what is the type of $M_1 \oplus M_2$ with respect to the direct sum of the implicit filtrations?

Aufgabe 5:

(*) For an ideal $I \subset K[\underline{x}]$, give an expression of the type (d,e) of $K[\underline{x}]/I$ in terms of the geometry of the variety defined by I. Conjectural expressions also welcome.