Singularities, Monodromy and Zeta Functions
Blatt 6

Exercises for discussion in the exercise class on 29.11.2018

Aufgabe 1:

Suppose $X \subseteq \mathbb{Q}_p^n \times \mathbb{Z}^m$ and let $\pi : \mathbb{Q}_p^n \times \mathbb{Q}_p^m \to \mathbb{Q}_p^n \times \mathbb{Z}^m$ be the map $(\bar{x}, \bar{y}) \mapsto (\bar{x}, v(\bar{y}))$. Here $v(\bar{y})$ is notation for the tuple of valuations of the components of $\bar{y}$.

Show that $\pi^{-1}(X)$ is a semi-algebraic subset of $\mathbb{Q}_p^n \times \mathbb{Q}_p^m$ if and only if $X = \pi(Y)$ for some semi-algebraic $Y \subseteq \mathbb{Q}_p^n \times \mathbb{Q}_p^m$.

Aufgabe 2:

Show that the map $\{x \in \mathbb{Q}_p^\times : 2 \mid v(x)\} \to \mathbb{Z}$ given by $x \mapsto \frac{v(x)}{2}$ is a semi-algebraic map.

Aufgabe 3:

Assume that $f : \mathbb{Q}_p \to \mathbb{Z}$ is semi-algebraic. Does it follow that $x \mapsto \lfloor \frac{f(x)-3}{5} \rfloor$ is a semi-algebraic function on $\mathbb{Q}_p$?

Course website: http://reh.math.uni-duesseldorf.de/~internet/Zeta_WS18/