Singularities, Monodromy and Zeta Functions
Blatt 11

Exercises for discussion in the class on 24.1.2019

Given $f \in \mathbb{Z}[x_1, ..., x_n]$ let $b_f \in \mathbb{Q}[s]$ be the Bernstein polynomial for f.

Aufgabe 1:

(a) Suppose that f is non-constant. Prove that $b_f(s)$ is divisible by $(s+1)$.
Hint: consider $s = -1$.

(b) What is b_f when f is constant?

Aufgabe 2:

Show that for any non-zero $a \in \mathbb{Z}$,

$$b_f = b_{af}.$$

Aufgabe 3:

Suppose $g = (f \circ A)$ for some $A \in \mathrm{GL}_n(\mathbb{Q})$.
Show that b_f is also the Bernstein polynomial for g.

Aufgabe 4:

(*) Suppose that f is non-constant and that there is no $a \in \mathbb{C}$ such that

$$f(a) = \frac{\partial f}{\partial x_1}(a) = \cdots = \frac{\partial f}{\partial x_n}(a) = 0.$$

Prove that $b_f = (s+1)$.
Hint: Use Hilbert's Nullstellensatz to find polynomials a_i such that

$$a_0(x)f(x) + \sum a_i(x)\frac{\partial f}{\partial x_i} = 1.$$

Then use those a_i to specify a differential operator P (satisfying $Pf^{s+1} = b_f f^s$) explicitly.

Aufgabe 5:

(*) (Warm-up for next lecture)
Consider f as a function from $\mathbb{C}^n$ to $\mathbb{C}$.
Suppose x is a point of the variety defined by f such that $f(x) = 0$ and grad $f(x) \neq 0$. Show there exist neighbourhoods $U \subseteq \mathbb{C}^n$ containing x and $D \subseteq \mathbb{C}$ containing 0 so that the restriction of f to $U \cap f^{-1}(D)$ is a topologically trivial fibration.