

# Talks at the workshop on zeta functions of groups and dynamical systems

## Bivariate zeta functions of finitely generated nilpotent groups

PAULA LINS DE ARAUJO – Universität Bielefeld

We introduce two bivariate zeta functions of a large class of finitely generated nilpotent groups encoding, respectively, the numbers of finite dimensional irreducible complex representations and the numbers of conjugacy classes of each cardinality of congruence quotients of the associated groups. We discuss some applications of these zeta functions, as well as arithmetic and analytic properties of their local factors.

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## Representation zeta functions of $p$ -adic analytic groups with non-vanishing radical

JAN MORITZ PETSCHICK – Heinrich-Heine-Universität Düsseldorf

I will give a progress report on my aims and endeavours of calculating the representation zeta functions of a  $p$ -adic analytic group with a non-vanishing radical.

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## The faithful dimension of $\exp(\mathfrak{f}_{n,c}(\mathbb{F}_p))$

ELENA TIELKER – Universität Bielefeld

The faithful dimension of a group  $\mathcal{F}$  is defined as the smallest integer  $n$  such that  $\mathcal{F}$  embeds into  $\mathrm{GL}_n(K)$  for some field  $K$ . Computing the faithful dimension of a group has applications for example in the theory of essential dimension. The groups we are interested in are of the form  $\mathcal{F}_p = \exp(\mathfrak{f} \otimes_{\mathbb{Z}} \mathbb{F}_p)$ , where  $\mathfrak{f}$  denotes a nilpotent  $\mathbb{Z}$ -Lie algebra of finite rank and  $\mathcal{F}_p$  is the  $p$ -group associated to  $\mathfrak{f} \otimes_{\mathbb{Z}} \mathbb{F}_p$  by the Lazard correspondence. Bardestani, Mallahi-Karai and Salmasian proved that under several assumptions the faithful dimension of those groups can be expressed as the solution of a rank minimization problem of the commutator matrix corresponding to  $\mathfrak{f}$ . Finally we apply this result to compute the faithful dimension of the  $p$ -group,  $p \geq 5$ , related to the free  $\mathbb{F}_p$ -Lie algebra on  $n$  generators and nilpotency class 4.

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## Reidemeister zeta functions of almost-crystallographic groups

SAM TERTOOUY – KU Leuven Campus Kulak Kortrijk

It is known that Reidemeister zeta functions of almost-Bieberbach groups are always rational. For almost-crystallographic (AC) groups, the rationality is as of yet unknown. In this talk, I will discuss some methods to determine which AC-groups admit Reidemeister zeta functions, as well as prove the rationality for AC-groups up to dimension 3.

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# Enumerating characters and conjugacy classes of finite p-groups

CHRISTOPHER VOLL – Universität Bielefeld

## On some Orbit Dirichlet series

CHRISTOPHER VOLL – Universität Bielefeld

Both talks will proceed in the time slot on Thursday, 11.15, and will be independent from each other.

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## Dynamical zeta functions of Reidemeister type

MALWINA ZIĘTEK – Uniwersytet Szczeciński

Let  $G$  be a group and  $\phi : G \rightarrow G$  an endomorphism. We call two elements  $\alpha, \alpha' \in G$ ,  $\phi$ -conjugate iff there exist  $\gamma \in G$  such that  $\alpha' = \gamma\alpha\phi(\gamma)^{-1}$ . The number of  $\phi$ -conjugacy classes is called *Reidemeister number* of  $\phi$  and is denoted by  $R(\phi)$ . We define the Reidemeister zeta function of  $\phi$  as follows:

$$R_\phi(z) = \exp\left(\sum_{n=1}^{\infty} \frac{R(\phi^n)}{n} z^n\right).$$

We assume that  $R(\phi^n) < \infty$  for all  $n > 0$ . We investigate for which groups and endomorphisms the Reidemeister zeta function is rational and when it has a functional equation. Moreover, we consider Reidemeister type zeta functions defined on unitary dual space.

## References

- [BMS17] M. Bardestani, K. Mallahi-Karai and H. Salmasian, Kirillov's orbit method and the polynomiality of the faithful dimension of p-groups, Preprint, arXiv:1712.02019v1, 2017.