

# Advanced Group Theory Seminar - WiSe 2024/25

## Braid groups

Braid groups are well known for being studied in many different areas of Mathematics thanks to the variety of approaches one can use to define them.

During the seminar we see most of these approaches, and we learn how they were used through the years to prove relevant properties such as torsion-freeness, hopfianity and residual finiteness and furthermore, to answer the word and conjugacy problems.

For our purposes, we follow the notes by Gonzalez-Meneses [1], which cover almost all the basics and collect often multiple proofs of the same results, so to gain a parallel view over all the approaches.

Due to the conciseness of such notes, I suggest to integrate the material -whenever required- with Chapter 1 of Kassel-Turaev's textbook [2] and/or follow the references already cited in [1].

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- Week 1. Braids and braid diagrams. See [2] (Sections 1.2.1, 1.2.2, 1.2.4).  
Braid groups, configuration spaces and mapping class groups. See [1] (Sections 1.1-1.5).
- Week 2. Braids as automorphisms of the free group. See [1] (Section 1.6) and [2] (Sections 1.5).  
First solution to the word problem. See [1] (Section 1.6.1).  
Residual finiteness and Hopfianity. See [1] (Section 1.6.2) and [2] (Sections 1.3.1, 1.3.2).
- Week 3. [1] Braid groups are torsion free. See [1] (Section 2) and [2] (Corollaries 1.18 and 1.29)  
Presentations of braid groups. See [1] (Section 3).
- Week 4. Garside structures. See [1] (Sections 4.1-4.2).  
Center of braid groups. See [1] (Section 4.3) and [2] (Section 1.3.3)  
Conjugacy problem. See [1] (Section 4.4)
- Week 5. TBA - depending on how far we got or what we would like to delve deeper.  
Week 6. TBA

### REFERENCES

- [1] J. González-Meneses, *Basic results on braid groups*, Ann. Math. Blaise Pascal **18** (2011), no. 1, 15–59, DOI 10.5802/ambp.293.  
[2] C. Kassel and V. Turaev, *Braid groups*, Graduate Texts in Mathematics, vol. 247, Springer, New York, 2008. With the graphical assistance of Olivier Dodane.

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