NEW TRENDS AROUND PROFINITE GROUPS

a CIRM Conference
in Levico Terme (Trentino)
13–17 September 2021

PROGRAMME

Schedule for the Conference “New Trends Around Profinite Groups 2021”

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
7:30--9:00	BREAKFAST				
9:00--9:30	Free Discussions				
9:30--10:30	Jaikin-Zapirain	Efrat	Tent	Kielak	Meiri
10:30--11:00	COFFEE BREAK				
11:00--12:00	Corob Cook	Quadrelli	Kionke	Castellano	Garrido
12:00--13:00	Nikolov	Maire	Segal	Kropholler	Schlage-Puchta
13:00--14:00	LUNCH				
14:00--16:00	Free Discussions		DIY EXCURSION(S)	Free Discussions	DEPARTURE
16:00--16:15	Trost	Zalesskii		Kochloukova	
16:15--16:30	Maglione				
16:30--16:45	Rajeev				
16:45--17:00					
17:00--17:30	Break			Break	
17:30--17:45	Tijsma	Di Gravina		Varghese	
17:45--18:00	Zordan	Reid		Petschick	
18:00--18:15	de las Heras	Macedo Lins de Araujo		Blumer	
18:15--18:30	Noce	Santos Rego			
18:30--18:45	Break			Break	
18:45--19:15	Discussion Session	Discussion Session		Discussion Session	
19:15–20:00	Free Discussions			Free Discussions	
20:00–21:00	DINNER				

S. Blumer Università degli Studi di Milano-Bicocca

Oriented right-angled Artin pro-p groups and Galois cohomology

In the context of Galois Theory, the most important result In the last decades is the proof of the Bloch–Kato Conjecture, mainly due to Rost and Voevodsky (2011). Nowadays there are several conjectures relying on that, whose aim is to classify the pro-p groups occurring as absolute Galois pro-p groups of certain fields via some cohomological characterization. In a joint work with C. Quadrelli and Th. Weigel, we considered a certain class of groups generalizing the pro-p completion of classical RAAGs; our study corroborates those conjectures, and, on the other hand, it provides a huge class of pro-p groups that do not occur as absolute Galois groups. In fact, the class of the discrete right-angled Artin groups plays an important role in group theory and geometric group theory, since it generalizes both the classes of the free groups and of the free-abelian groups, and it provides a large test bed for those conjectures.
In the poster I listed some of the main definitions and conjectures in Galois Theory as well as the outcomes we achieved.

I. Castellano Università degli Studi di Milano-Bicocca

Euler characteristic and zeta functions for TDLC-groups

The Euler–Poincaré characteristic of a discrete group is an important (but also quite mysterious) invariant. It is usually just an integer or a rational number and reflects many quite significant properties. We will show that the realm of totally disconnected locally compact groups admits an analogue of the Euler–Poincaré characteristic which surprisingly is no longer just an integer, or a rational number, but a rational multiple of a Haar measure. What arouses our curiosity is also the fact that – in some cases – the Euler–Poincaré characteristic turns out to be miraculously related to a zeta-function. Joint work with Gianmarco Chinello and Thomas Weigel.

G. Corob Cook Cambridge

Counting irreducible modules for profinite groups

We say a profinite group G has UBERG if the number of irreducible G-modules of order k grows polynomially in k. This is equivalent to the completed group ring $\hat{\mathbb{Z}}[[G]]$ being generated with positive probability by n random elements, for some n (with respect to the Haar measure). I will talk about recent work, joint with S. Kionke and M. Vannacci, where we give algebraic conditions for G to have UBERG in terms of the sizes of the crown-based powers of monolithic primitive groups appearing as a quotient of G. As an application, we show that UBERG is not closed under extensions, unlike G being positively finitely generated (PFG), but UBERG-by-PFG groups do have UBERG. I will also discuss our work on a probabilistic version of the type FP1 condition, and some examples showing how these conditions relate to each other and to the PFG condition.

I. de las Heras Heinrich-Heine-Universität Düsseldorf

Powerfully solvable and powerfully simple groups

For the purpose of bringing some order to the huge category of finite groups, one usually classifies them into different subfamilies, such as simple groups, nilpotent groups, solvable groups, etc. In the (still huge) family of finite p-groups, though, this classification makes no sense in general, as it is not tailored to the particular group structure of the finite p-groups. In the family of powerful groups, however, one can consider subfamilies in an analogous way as for general finite groups by assigning the role that the (normal) subgroups have in the context of general finite groups, to the powerful (powerfully embedded) subgroups of powerful groups.
In this talk, we will briefly introduce three of these families, namely, the powerfully nilpotent, the powerfully solvable and the powerfully simple groups. This is joint work with Gunnar Traustason.

L. M. di Gravina Università degli Studi di Milano-Bicocca

The Möbius function of subgroup lattices

If G is a finitely generated profinite group, there is a connection between the probabilistic zeta function of G and the Möbius function defined on the lattice of the open subgroups of G. In particular, if G is positively finitely generated, some questions arise about the growth of the absolute value of the Möbius function of G and the number of subgroups of G with non-zero value for this function.

I. Efrat Ben-Gurion University of the Negev

Filtrations of profinite groups as intersections and absolute Galois groups

The general structure of absolute Galois groups of fields as profinite groups is still a mystery. Among the very few known properties of such groups are several "Intersection Theorems", describing subgroups in standard filtrations of absolute Galois groups as the intersection of all normal open subgroups with quotient in a prescribed list of finite groups. These theorems are based on deep cohomological properties of absolute Galois groups. We will present a general "Transfer Theorem" for profinite groups, which explains what lies behind these intersection theorems.

A. Garrido Universidad Autónoma de Madrid

On various profinite completions of groups acting on rooted trees

Groups that act faithfully on rooted trees can be completed in different ways to obtain profinite groups. The profinite completion of the group maps onto each of them. Determining the kernels of these maps is known as the congruence subgroup problem. This has been studied by various authors over the last few years, most notably for self-similar groups and (weakly) branch groups. In the case of self-similar regular branch groups, much insight can be gained into this problem using a symbolic-dynamical point of view. After reviewing the problem and previous work on it, I will report on work in progress with Zoran Šunić on determining the dynamical complexity of these completions and calculating some of these kernels with relative ease.
Examples will be given. No previous knowledge of self-similar or branch groups is required.

A. Jaikin-Zapirain Universidad Autónoma de Madrid

The finite and the soluble genus of a finitely generated free group

A well-known question of Remeslennikov asks if finitely generated residually finite groups with free profinite completion are free. I will show that such groups should be residually nilpotent (and so, parafree). I will also show that the same conclusion holds for finitely generated residually-(finite soluble) groups with free prosoluble completion.

D. Kielak University of Oxford

Finite-index subgroups of one-relator groups

We will look at the following two questions: what can be said about a one-relator group whose all finite-index subgroups are also one-relator? Which one-relator groups are good in the sense of Serre?

S. Kionke — FernUniversität in Hagen

Amenability and profinite completions of finitely generated groups

Is it possible to decide whether or not a residually finite group is amenable by looking at its finite quotients only? In general, the answer is negative. In this talk we exhibit an uncountable family of finitely generated counterexamples based on the theory of branch groups. On the other hand, we explain why uniform amenability, a stronger concept introduced in the 70s, can be detected from the finite quotients of residually finite groups.
This is based on joint work with Eduard Schesler.

D. Kochloukova — Universidade Estadual de Campinas (UNICAMP)

Subdirect product of limit groups over Droms RAAGs

This is a joint work with Jone Lopez de Gamiz, available online on arxiv. We generalize the existing theory of subdirect products of limits groups (over free groups) to the case of subdirect products of limit groups over Droms RAAGs. Here RAAG means right angled Artin group and a Droms RAAG is a special one such that every finitely generated subgroup is a RAAG again. This condition can be checked by the underlying graph : it should not contain an embedded square or a line of length 3. Our work is based on some recent results (both available on arxiv) on limit groups over coherent RAAGs due to M. Casals-Ruiz, A. Duncan, I. Kazachkov and on the structure theory of the limit groups over Droms RAAGs due to Jone Lopez de Gamiz.

P. Kropholler — University of Southampton

Condensed mathematics: could this be the new trend?

Its an old problem or frustration in the theory of topological groups or topological abelian groups that the First Isomorphism Theorem is not true: the problem is that a bijective morphism from one topological group to another is not necessarily an isomorphism because it is not necessarily a homeomorphism: that is to say, its inverse might be discontinuous. Scholze, Clausen, Barwick and others have been laying foundations for a new condensed mathematics and it looks as though it should be useful in the development of homological algebra in the context of TDLC groups and modules that carry topologies. The basic idea is to replace topological spaces by sheaves on a pro-etale site and the main reason why this idea is promising is that it leads rather quickly to abelian categories. In particular this provides a framework in which the first isomorphism theorem works again. Could this become a new trend? Lets see how some basic aspects of the machinery will need to be pieced together.

P. Macedo Lins de Araujo — Katholieke Universiteit Leuven

Reidemeister numbers of topological automorphisms

Twisted conjugacy of discrete groups has been widely investigated in the past decades. On this short presentation, we discuss the profinite version of twisted conjugacy classes and the Reidemeister numbers.

J. Maglione — Universität Bielefeld

Flag Hilbert–Poincaré series and Igusa zeta functions of hyperplane arrangements

We define a class of multivariate rational functions associated with hyperplane arrangements called flag Hilbert–Poincaré series, and we show how these rational functions are connected to Igusa zeta functions. We report on a general self-reciprocity result and explore other connections within algebraic combinatorics. This is joint work with Christopher Voll.

C. Maire Université de Franche Comté

On Galois representations with large image

Let $G_{\mathbb{Q}}$ be the absolute Galois of $\mathbb{Q}$. For every prime number $p \geq 3$ and every integer $m \geq 1$, we show the existence of a continuous Galois representation $\rho : G_{\mathbb{Q}} \to \mathrm{Gl}_m(\mathbb{Z}_p)$ which has open image and is unramified outside $\{p, \infty\}$ (resp. outside $\{2, p, \infty\}$) when $p \equiv 3 \bmod 4$ (resp. $p \equiv 1 \bmod 4$). To do that, we combine some properties of pro-p-extensions of number fields with restricted ramification, and lifting mod p Galois representations in terms of embedding problems.

C. Meiri Technion - Israel Institut of Technology

Is being a higher rank lattice a first order property?

We will discuss joint work with Nir Avni which gives some evidence to the following conjecture: There is a first order sentence φ in the language of groups such that, for every finitely generated group Δ, the sentence φ holds in Δ if and only if Δ is an irreducible lattice in a higher rank semisimple group.

N. Nikolov University of Oxford

Profinite groups with positive rank gradient

I will introduce the concept of rank gradient with emphasis on profinite groups, will discuss some open questions and report on recent progress. In particular I will outline the proof that a profinite group G with positive rank gradient does not satisfy a group law. When G is a pro-p group the argument is easier and proves that G contains a dense free subgroup.

M. Noce Georg-August-Universität Göttingen

Ramification structure in quotients of Grigorchuk groups

Groups of automorphisms of regular rooted trees have been studied for years as an important source of groups with interesting properties. For example, the Grigorchuk groups provide a family of groups with intermediate word growth and the torsion Grigorchuk groups constitute a counterexample to the General Burnside Problem. For these groups there is a natural family of normal subgroups of finite index, which are the level stabilizers. The goal of this talk is to show that the quotients by such subgroups admit a ramification structure. Roughly speaking, groups of surfaces isogenous to a higher product of curves are characterised by the existence of a ramification structure. Recall that an algebraic surface S is isogenous to a higher product of curves if it is isomorphic to $(C_1 \times C_2)/G$, where C_1 and C_2 are curves of genus at least 2, and G is a finite group acting freely on $C_1 \times C_2$. In this talk, we first introduce the Grigorchuk groups and then we show that their quotients admit ramification structures, providing the first explicit infinite family of 3-generated finite 2-groups with ramification structures that are not Beauville. This is joint work with A. Thillaisundaram.

J. M. Petschick Heinrich-Heine-Universität Düsseldorf

Conditions for constant spinal groups to be periodic

Famously, the Gupta–Sidki p-groups are examples of infinite finitely generated periodic groups. Over the past decades, various generalisations of these groups have been defined, some of which are and some of which are not periodic groups. I will explain what a constant spinal groups is, and present conditions ensuring that such a group is periodic.

C. Quadrelli Università degli Studi di Milano-Bicocca

Who wants to be an absolute Galois group?

Detecting which profinite groups occur as absolute Galois groups (AGGs for short) of fields is one of the main open problems in Galois theory. The celebrated Bloch–Kato Conjecture — proved 10 years ago by V. Voevodsky and now called the Norm Residue Theorem — gives a description of the Galois cohomology of absolute Galois groups in terms of low-degree cohomology. This achievement provided new tools to study the structure of AGGs and their maximal pro-p groups: e.g., such pro-p groups have quadratic Galois cohomology (i.e., they are *Bloch–Kato* pro-p groups) where some Massey products vanish, and moreover they satisfy a formal version of Hilbert90 (i.e., they are *1-cyclotomic*, or *hereditarily Kummerian*, pro-p groups). In the talk we shall see how "powerful" each of these three properties is, and some concrete examples of pro-p groups which do not satisfy (some of) these properties — and which are new examples of pro-p groups not occurring as AGGs.
Some of the results which will be mentioned were obtained in collaboration with (various subsets of): S. Blumer, A. Cassella, I. Efrat, and Th. Weigel.

K. Rajeev Heinrich-Heine-Universität Düsseldorf

Representations of GGS groups

Grigorchuk–Gupta–Sidki (GGS) groups are generalisations of the second Grigorchuk group and the Gupta–Sidki groups. We will investigate the asymptotic distribution of irreducible complex representations of GGS groups.

C. Reid University of Newcastle (Australia)

Totally disconnected locally compact groups with just infinite locally normal subgroups

I will be talking about some properties of totally disconnected, locally compact (t.d.l.c.) groups that are locally isomorphic to a finite direct product of just infinite profinite groups. The main goals are to give a converse (in the profinite case) to a 1971 theorem of J. Wilson about near-complements in just infinite groups, and to build upon a 2011 theorem of Barnea–Ershov–Weigel on the abstract commensurator of a hereditarily just infinite profinite group, giving a sufficient condition to obtain a nondiscrete simple locally normal subgroup.

Y. Santos Rego Otto-von-Guericke-Universität Magdeburg

Is property R_∞ a profinite property?

A group has property R_∞ if all its automorphisms have infinitely many twisted conjugacy classes. While there are multiple works dedicated to check which of our favorite groups exhibit R_∞, the property itself remains rather mysterious. In this short talk we shall discuss the following question: is R_∞ witnessed by finite quotients?

J.-C. Schlage-Puchta Universität Rostock

On certain growth types

The first question when facing an asymptotic invariant is to compute this invariant for a variety of interesting examples. Once a certain pool of computations exists, the reverse question arises naturally: What are the possible values for this invariant, and what are the possible values, if the objects to which the invariants are attached are restricted in some natural way?
In this talk I will present some recent results in this direction, concerning the subgroup growth of pro-p groups, the normal growth of profinite groups, and the orbit growth of discrete groups.
This talk contains joint work with Yiftach Barnea, Benjamin Klopsch, and Leyli Jafari.

D. Segal All Souls College Oxford

Axiomatizability for profinite groups

There is a kind of tension between first-order logic and algebra, as it is not easy to talk about about subsets. However, it can be interesting to explore the interaction; for example, to determine which algebraic properties can, and which can't, be expressed in first-order language.
'Being isomorphic to a particular group' is one such property. We say that a group G is *axiomatizable* in a class $\mathcal{C}$ if G is the only member of $\mathcal{C}$ (up to isomorphism) that satisfies the same sentences as G, and *finitely axiomatizable* if there is a single sentence σ such that G is the unique member of $\mathcal{C}$ satisfying σ. Now we ask: exactly which groups are (finitely) axiomatizable?
General principles of model theory immediately exclude certain kinds of group, and some famous hard-core geometric group theory excludes others; but - surpringly, perhaps - quite a lot of groups do have one of these properties.
The subject has some history when $\mathcal{C}$ is the class of all finitely generated groups. I will talk about the case where $\mathcal{C}$ is the class of all profinite groups. We have some suggestive results, but the topic is largely unexplored and there are many questions to answer.

K. Tent Westfälische Wilhelms-Universität Münster

Defining R and $G(R)$

In joint work with Segal we use the fact that for Chevalley groups $G(R)$ of rank at least 2 over a ring R the root subgroups are (nearly always) the double centralizer of a corresponding root element to show under mild restrictions on the ring R that R and $G(R)$ are bi-interpretable. (This holds in particular for any field k.) For such groups it then follows that the group $G(R)$ is finitely axiomatizable in the appropriate class of groups provided R is finitely axiomatizable in the corresponding class of rings. This specializes in particular to the profinite setting.

D. Tijsma Heinrich-Heine-Universität Düsseldorf

Finite order elements in the Nottingham group

The Nottingham group at a prime p is the group of (formal) power series $t + a_2t^2 + a_3t^3 + \dots$ in the variable t with coefficients a_i from the field with p elements, where the group operation is given by composition of power series. Only a handful of power series of finite order are explicitly known through a formula for their coefficients. I will argue that it is advantageous to describe such series in closed computational form through automata, based on effective versions of proofs of Christol's theorem identifying algebraic and automatic series. Using this new method, we have found, in the case of $p = 2$, several new closed form formulas for series of finite order 4 and some automatic descriptions of series of order 8. This is joint work with Jakub Byszewski and Gunther Cornelissen.

A. Trost Ruhr Universität Bochum

Strong boundedness of $\mathrm{SL}_2(R)$ for rings of S-algebraic integers with infinitely many units

In this talk, I will give a description of the minimal speed with which a collection of finitely many conjugacy classes can generate the group $\mathrm{SL}_2(R)$ for R the ring of all S-algebraic integers in a number field K such that R has infinitely many units. I will briefly discuss the corresponding proofs involving model theoretic compactness arguments and a result due to Costa and Keller about the normal subgroups of $\mathrm{SL}_2(R)$. If time allows I will also explain how certain bad prime ideals of the ring R obstruct the existence of small collections of conjugacy classes generating the group $\mathrm{SL}_2(R)$.

O. Varghese Otto-von-Guericke-Universität Magdeburg

Automatic continuity from a geometric perspective

We study abstract group actions of locally compact Hausdorff groups on CAT(0) cube complexes. Under mild assumptions on the action we show that it is continuous or has a global fixed point. As a consequence we obtain a geometric proof for the fact that any abstract group homomorphism from a locally compact Hausdorff group into an Artin–Tits group of FC-type is continuous.

P. Zalesskii Universidade de Brasília

Splitting of pro-p groups (as an amalgam or HNN)

I shall present Stallings type results of splittings of pro-p groups as an amalgamated free pro-p product or a pro-p HNN-extension as well as some other aspects of such splittings.

M. Zordan Imperial College London

Rationality of representation zeta functions of compact p-adic analytic groups

This talk is a short survey on how model theoretic arguments may be used to prove that the representation zeta function of certain compact p-adic analytic groups is (virtually) rational. I will focus on the role played by projective representations and cohomology groups.

Registered participants should have received the relevant information for joining the scheduled conference activities online; if you did not receive this information or misplaced it and need assistance, please contact Augusto Micheletti from CIRM (`micheletti@fbk.eu`).

version – 12 September 2021