

# INTRODUCTION TO ZETA FUNCTIONS OF GROUPS AND DYNAMICAL SYSTEMS

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In preparation for the September workshop on zeta functions of groups and dynamical systems we will have “sneak preview” of some basic definitions relevant to the themes of the mini-courses.

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**TALK 1: Reidemeister zeta functions of groups.** *Chapter 2.4 in [3].*

The Reidemeister number and zeta function of an endomorphism of a group, counting twisted conjugacy classes, will be defined and calculated for the case of finite groups, where the zeta function is rational.

*Plan:* Proof the results of Chapter 2.4. in [3] (or 1.4 in [4]). Give an overview of the other results in Chapter 2 of [3].

**TALK 2: Nielsen Fixed Point Theory.** *Chapter 1 of [5].*

The Reidemeister and Nielsen numbers of a continuous map count the classes of (essential) fixed points “up to homotopy”. They will be defined and the Nielsen fixed point theorem, a quantitative generalization of Brouwers fixed point theorem, will be shown.

*Plan:* Follow the definitions of I.§1 – I.§4 of [5]. Sketch a proof the Nielsen fixed point theorem.

**TALK 3: Infra-Nilmanifolds.**

Infra-nilmanifolds generalize flat manifolds. Infra-nilmanifolds, their endomorphisms, and almost-crystallographic groups will be defined, along with examples.

*Plan:* Give the definitions along [1] or [2] and expand on examples.

## References

- [1] Karel Dekimpe. WHAT IS... *an Infra-nilmanifold Endomorphism?* Notices Amer. Math. Soc. 58 (2011), no. 5, 688–689.  
<http://www.ams.org/notices/201105/rtx110500688p.pdf>
- [2] Karel Dekimpe. A Users’ Guide to Infra-nilmanifolds and Almost–Bieberbach groups.  
<https://arxiv.org/pdf/1603.07654.pdf>
- [3] Alexander Fel’shtyn; Richard Hill. *Dynamical Zeta Functions, Nielsen Theory and Reidemeister Torsion.* Mem. Amer. Math. Soc. 147 (2000), no. 699,  
<https://arxiv.org/pdf/chao-dyn/9603017.pdf>
- [4] Alexander Fel’shtyn; Richard Hill. *The Reidemeister zeta function with applications to Nielsen theory and a connection with Reidemeister torsion.* K-Theory 8 (1994), no. 4, 367–393.  
[https://www.researchgate.net/publication/226295799\\_The\\_Reidemeister\\_zeta\\_function\\_with\\_applications\\_to\\_Nielsen\\_theory\\_and\\_a\\_connection\\_with\\_Reidemeister\\_torsion](https://www.researchgate.net/publication/226295799_The_Reidemeister_zeta_function_with_applications_to_Nielsen_theory_and_a_connection_with_Reidemeister_torsion)
- [5] Boju Jiang. *Lectures on Nielsen Fixed Point Theory.* Contemporary Mathematics, 14. American Mathematical Society, Providence, R.I., 1983.