

Klausur B Wiwi II

Aufgabe 1

a) richtig

b) falsch

c) richtig

d) falsch

e) richtig

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Aufgabe 2 $x \in (0, \infty)$, $f(x) = x e^{-x^2}$

a) $\lim_{x \rightarrow 0} f(x) = \underline{0}$ 1P

$\lim_{x \rightarrow \infty} f(x) = \underline{0}$ 1P

b) $f'(x) = \underline{(1 - 2x^2)e^{-x^2}}$ 2P

$\varepsilon_f(x) = \frac{x f'(x)}{x e^{-x^2}} = \underline{1 - 2x^2}$ 2P

c) $|\varepsilon_f(x)| < 1 \quad \stackrel{x > 0}{\iff} \underline{x \in (0, 1)}$ 2P

d) $f'(x) \geq 0$?

$\iff 1 - 2x^2 \geq 0$

$\stackrel{x > 0}{\iff} \begin{array}{l} x \in (0, \frac{1}{\sqrt{2}}] \text{ monoton steigend} \\ x \in [\frac{1}{\sqrt{2}}, \infty) \text{ monoton fallend} \end{array}$

2P

e) Max bei $x = \frac{1}{\sqrt{2}}$ nach a) und d)

$f(\frac{1}{\sqrt{2}}) = \underline{\frac{1}{\sqrt{2}} e^{-\frac{1}{2}}}$ 1P

Infimum $\inf_{x \in (0, \infty)} f(x) = \underline{0}$ 1P

mit a) und $x e^{-x^2} > 0$
für $x \in (0, \infty)$

$$f) \quad f''(x) = (4x^3 - 6x)e^{-x^2} \quad \underline{2P}$$

$$g) \quad f''(x) \leq 0?$$

$$\stackrel{x > 0}{\iff} 4x^2 - 6 \leq 0$$

$$\stackrel{x > 0}{\iff} \underline{x \in (0, \sqrt{\frac{3}{2}}]} \quad \underline{2P}$$

progressiv fallend:

$$\& \quad \begin{aligned} f'(x) &\leq 0 \\ f''(x) &\leq 0 \end{aligned}$$

$$\rightarrow \underline{[\frac{1}{\sqrt{2}}, \infty) \cap (0, \sqrt{\frac{3}{2}}]} = \underline{[\frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}]} \quad \underline{2P}$$

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Aufgabe 3

$$f(x,y) = x^4 y^3$$

$$a) \quad \frac{\partial f}{\partial x}(x,y) = \underline{\underline{4x^3 y^3}}$$

1P

$$\frac{\partial f}{\partial y}(x,y) = \underline{\underline{3x^4 y^2}}$$

1P

$$\frac{\partial^2 f}{\partial x^2}(x,y) = \underline{\underline{12x^2 y^3}}$$

1P

$$\frac{\partial^2 f}{\partial y^2}(x,y) = \underline{\underline{6x^4 y}}$$

1P

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial^2 f}{\partial y \partial x}(x,y) = \underline{\underline{12x^3 y^2}}$$

1P

$$b) \quad \varepsilon_{f,x}(x,y) = \frac{x \frac{\partial f}{\partial x}(x,y)}{f(x,y)} = \underline{\underline{4}}$$

1P

$$\varepsilon_{f,y}(x,y) = \frac{y \frac{\partial f}{\partial y}(x,y)}{f(x,y)} = \underline{\underline{3}}$$

1P

Aufgabe 4

a) $F(x) = \log(1+x^2)$

2 P

b) $F(x) = -(1+x)e^{-x}$

$$\int_0^{\infty} f(x) dx = \lim_{R \rightarrow \infty} F(x) \Big|_0^R = 0 + e^0 = \underline{1} \quad \left. \vphantom{\int_0^{\infty} f(x) dx} \right\} \underline{4 P}$$

$$\begin{aligned} \text{c) } \frac{1}{6} \int_{-1}^5 (x-2)^2 dx &= \frac{1}{6} \cdot \frac{1}{3} (x-2)^3 \Big|_{-1}^5 \\ &= \frac{1}{6} \left(\frac{3^3}{3} + \frac{3^3}{3} \right) = \underline{\underline{3}} \quad \underline{3 P} \end{aligned}$$

d) $E_f(x) = \frac{x f'(x)}{f(x)} = x^2$

$$\Rightarrow \int \frac{f'(x)}{f(x)} dx = \int x dx$$

$$\Rightarrow \log(f(x)) = \frac{1}{2}x^2 + c$$

$$\Rightarrow \underline{\underline{f(x) = \tilde{c} e^{\frac{1}{2}x^2}}} \quad \underline{4 P}$$

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Aufgabe 5 $f(x,y) = 2x^3 - 3x^2 - 12x + (y^2 + 2y - 7)e^y$

a) $\nabla f(x,y)$ & kritische Stellen

$$\frac{\partial f}{\partial x}(x,y) = 6x^2 - 6x - 12$$

$$\frac{\partial f}{\partial y}(x,y) = (y^2 + 4y - 5)e^y$$

$$\frac{\partial f}{\partial x}(x,y) = 0 \iff \underline{x=2} \vee \underline{x=-1}$$

$$\frac{\partial f}{\partial y}(x,y) = 0 \iff \underline{y=1} \vee \underline{y=-5}$$

6 P

→ kritische Stellen

$$\begin{aligned} &(-1, 1), (2, 1) \\ &(-1, -5), (2, -5) \end{aligned}$$

b)

$$\text{Hess } f(x,y) = \begin{pmatrix} 12x - 6 & 0 \\ 0 & (y^2 + 6y - 1)e^y \end{pmatrix}$$

4 P

c)

$$\text{Hess } f(-1, 1) = \begin{pmatrix} -18 & 0 \\ 0 & 6e^1 \end{pmatrix} \text{ Sattelpunkt}$$

$$\text{Hess } f(2, 1) = \begin{pmatrix} 18 & 0 \\ 0 & 6e^1 \end{pmatrix} \text{ Minimum}$$

$$\text{Hess } f(-1, -5) = \begin{pmatrix} -18 & 0 \\ 0 & -6e^{-5} \end{pmatrix} \text{ Maximum}$$

$$\text{Hess } f(2, -5) = \begin{pmatrix} 18 & 0 \\ 0 & -6e^{-5} \end{pmatrix} \text{ Sattelpunkt}$$

8 P