Notation / Sprechweisen

Koeffizientenmatrix:

$$A = \begin{array}{cc} \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} & \begin{matrix} m \\ \text{Zeilen} \end{matrix} \\ n \ \text{Spalten} & \end{array}$$

$$= (a_{ij})_{\substack{i=1,\dots,m \\ j=1,\dots,n}}$$

$m \times n$ - Matrix

Zeile (row roman)

Spalte (column catholic)

erweiterte Koeffizientenmatrix

$$(A,b) = \left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & & & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right)$$

$$A \cdot \underline{x} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$:= \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n \end{pmatrix}$$

z. B.:

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad r = 0$$

$$A = \begin{array}{c} \\ \\ \end{array}\begin{array}{ccc} a_{11} & a_{22} & a_{33} \\ \end{array}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad r = 3$$

$$j_1 = 1,\ j_2 = 2,\ j_3 = 3$$

$$\begin{array}{ccccc} & a_{12} & & a_{24} & a_{35} \end{array}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad r = 3$$

$$j_1 = 2 \quad j_2 = 4 \quad j_3 = 5$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

$$k = 4$$

$$(A, b) = \left(\begin{array}{ccccccc|c} 2 & 3 & 0 & 0 & 4 & 6 & 5 & b_1 \\ 0 & 1 & 1 & 0 & 3 & 2 & 0 & b_2 \\ 0 & 0 & 3 & 0 & 0 & 0 & 1 & b_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \qquad \begin{array}{l} m = 4 \\ n = 7 \\ r = 3 \end{array}$$

$$x_4 = \lambda_1$$
$$x_5 = \lambda_2$$
$$x_6 = \lambda_3$$
$$x_7 = \lambda_4$$

(Zeile 3) $\left(0 \quad 0 \quad 3 \quad \overset{\neq 0}{0} \quad 0 \quad 0 \quad 1 \;\middle|\; b_3\right)$

$$3x_3 + \lambda_4 = b_3,$$

also $3x_3 + \lambda_4 = b_3$

also $$\boxed{x_3 = \left(\tfrac{1}{3}\right) b_3 + \left(\tfrac{-1}{3}\right) \lambda_4}$$

$c_{33} = \frac{1}{3}$ $\quad d_{34}$

$d_{31} = 0$

$d_{32} = 0$

$d_{33} = 0$

$d_{34} = -\frac{1}{3}$

$$\begin{array}{cccccccc} & & & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \\ (0 & 1 & 1 & 0 & 3 & 2 & 0 & \mid b_2) \end{array}$$

(Zeile 2)

$$x_2 + x_3 + 3\lambda_2 + 2\lambda_3 = b_2,$$

also $$x_2 + \left(\tfrac{1}{3} b_3 + \left(-\tfrac{1}{3}\right)\lambda_4\right) + 3\lambda_2 + 2\lambda_3 = b_2,$$

also $$x_2 = \underset{c_{22}=1}{\wedge} b_2 \underbrace{-\tfrac{1}{3}}_{c_{23}} b_3 \underbrace{-3}_{d_{22}} \lambda_2 \underbrace{-2}_{d_{23}} \lambda_3 + \underbrace{\tfrac{1}{3}}_{d_{24}} \lambda_4$$

(Zeile 1) $\begin{array}{cccccccc} & & & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \\ \big(2 & 3 & 0 & 0 & 4 & 6 & 5 & \big| \, b_1 \big) \end{array}$

$$2x_1 + 3x_2 + 4\lambda_2 + 6\lambda_3 + 5\lambda_4 = b_1,$$

also

$$2x_1 + 3\left(b_2 - \frac{1}{3}b_3 - 3\lambda_2 - 2\lambda_3 + \frac{1}{3}\lambda_4\right) + 4\lambda_2 + 6\lambda_3 + 5\lambda_4 = b_1,$$

also

$$x_1 = \frac{1}{2}\left[b_1 - 3b_2 + b_3 + 5\lambda_2 - 6\lambda_4\right],$$

also

$$x_1 = \frac{1}{2}b_1 - \frac{3}{2}b_2 + \frac{1}{3}b_3 + \frac{5}{2}\lambda_2 - 3\lambda_4$$

(a)

$$
\begin{aligned}
x_1 &= \tfrac{1}{2} b_1 - \tfrac{3}{2} b_2 + \tfrac{1}{3} b_3 + \tfrac{5}{2}\lambda_2 - 3\lambda_4 \\
x_2 &= b_2 - \tfrac{1}{3} b_3 - 3\lambda_2 - 2\lambda_3 + \tfrac{1}{3}\lambda_4 \\
x_3 &= \tfrac{1}{3} b_3 - \tfrac{1}{3}\lambda_4 \\
x_4 &= \lambda_1 \\
x_5 &= \lambda_2 \\
x_6 &= \lambda_3 \\
x_7 &= \lambda_4
\end{aligned}
$$

(b) ...

(c)

$$
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}
= \overbrace{\begin{pmatrix} \tfrac{1}{2} b_1 - \tfrac{3}{2} b_2 + \tfrac{1}{3} b_3 \\ b_2 - \tfrac{1}{3} b_3 \\ \tfrac{1}{3} b_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}^{\underline{v}}
+ \lambda_1 \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{\underline{w}_1}
+ \lambda_2 \cdot \begin{pmatrix} 5/2 \\ -3 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}
+ \lambda_3 \begin{pmatrix} 0 \\ -2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}
+ \lambda_4 \underbrace{\begin{pmatrix} -3 \\ 1/3 \\ -1/3 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}}_{\underline{w}_4}
$$