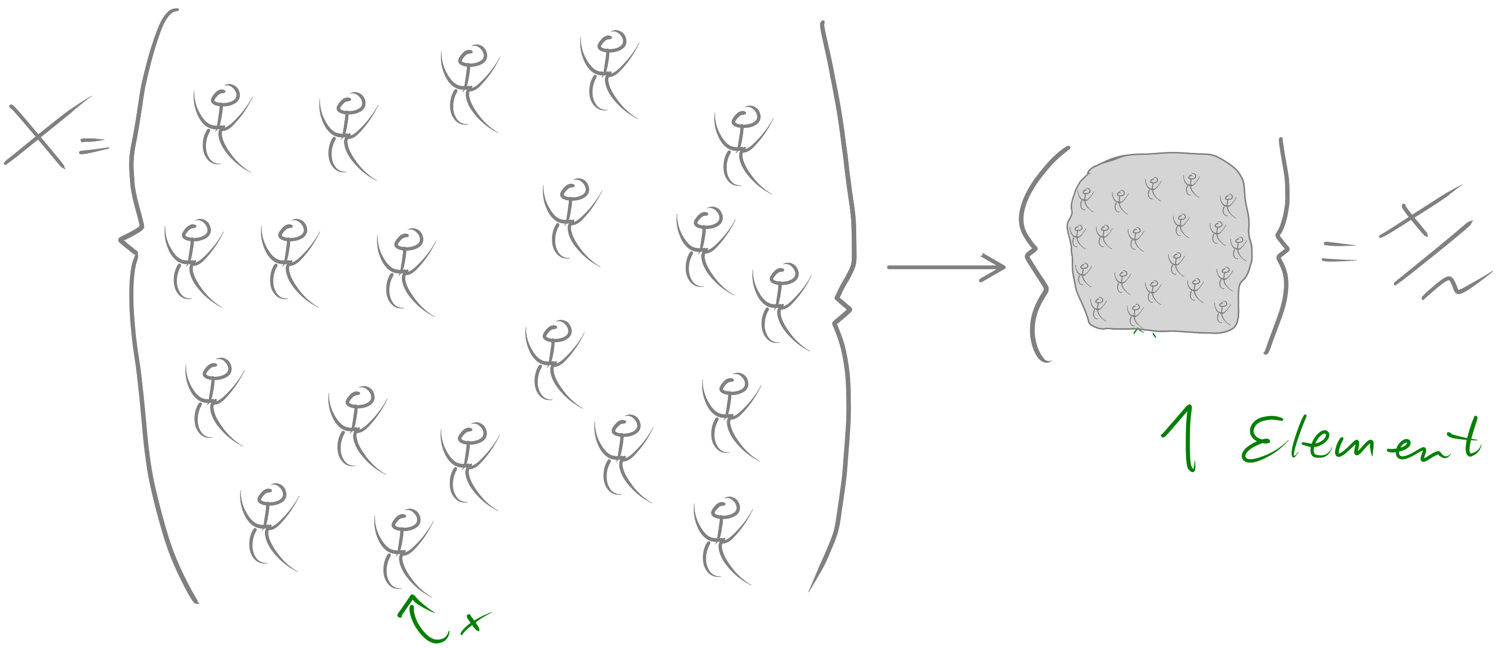
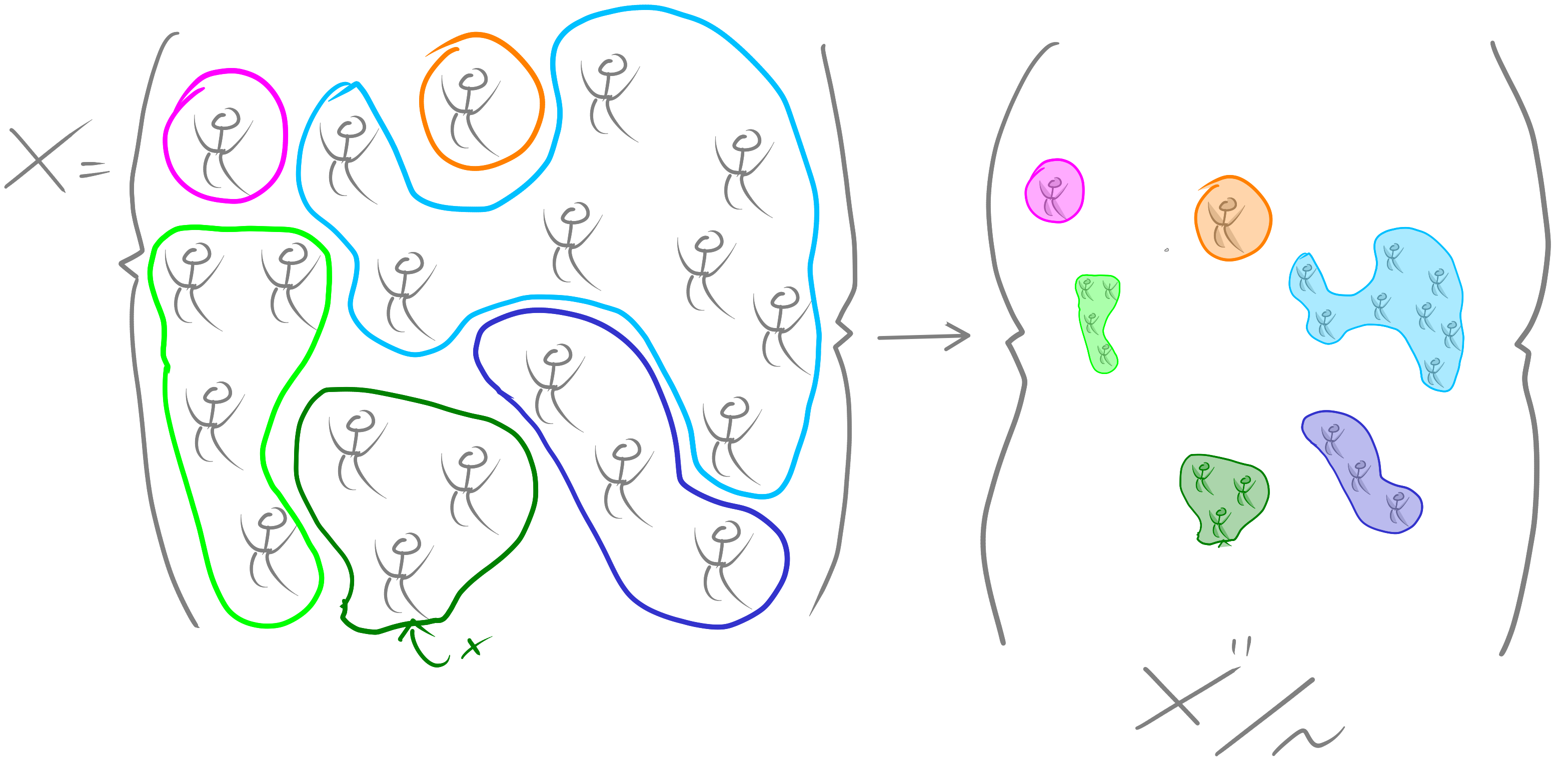
X = Weltbevölkerung

$x \sim y :\iff x$ ist mit y verwandt



$x \sim y \iff x$ und y haben dieselben Eltern



$x \sim y :\Longleftrightarrow$ x und y haben am selben Tag / Geburtstag

$[x] = \{$alle Menschen die am selben Tag wie x Geburtstag haben$\}$

$$X = \mathbb{R}^2 \qquad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} :\Longleftrightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

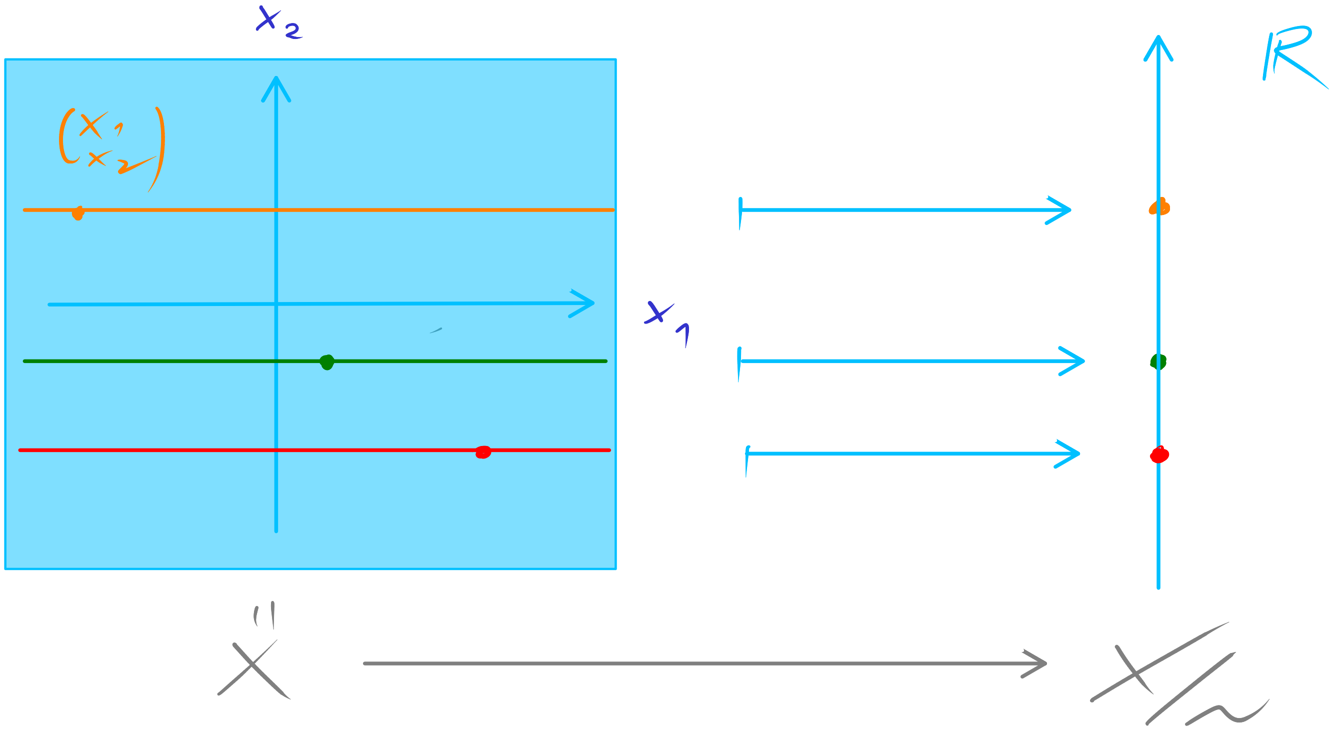
$$\left[\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right] = \left\{ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R}^2 \;\middle|\; \underline{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} \right\}$$
$$= \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right\}$$

$$\mathbb{R}^2 \xrightarrow[\text{bijektiv}]{} \mathbb{R}^2/\sim$$

$\underline{x}$

$\{\underline{x}\}$, für $\underline{x} \in \mathbb{R}^2$

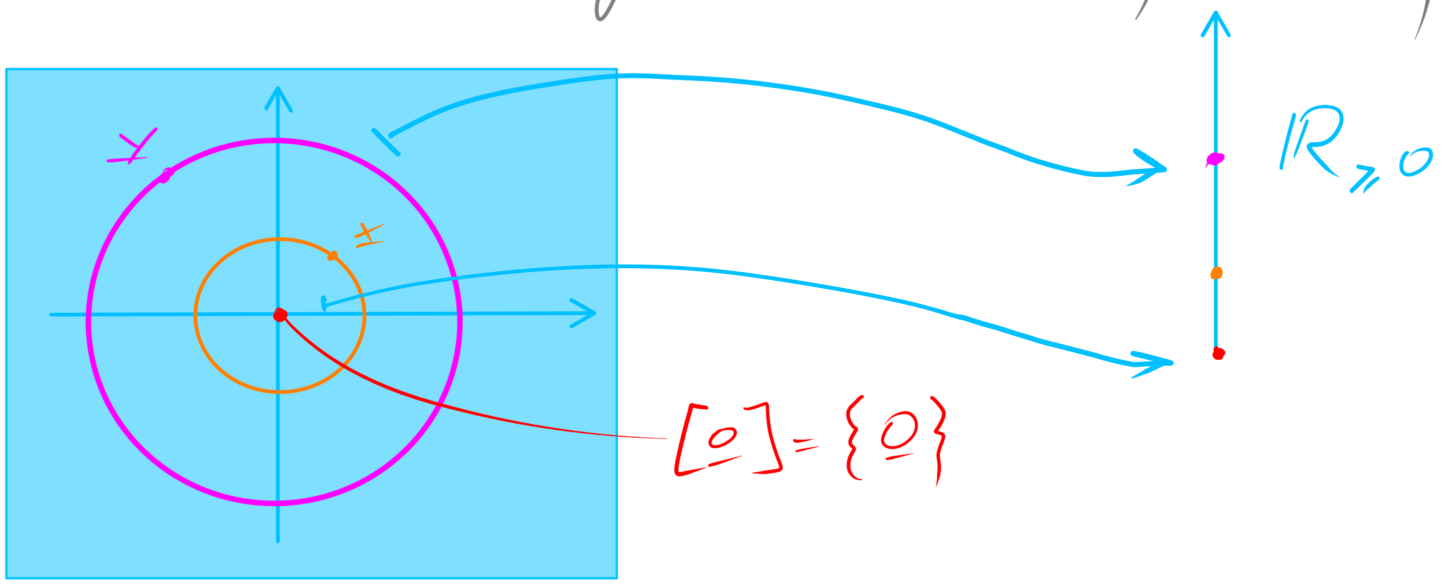
$X = \mathbb{R}^2 \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} :\Longleftrightarrow x_2 = y_2$



$[\underline{x}]$ = Parallele zur x_1-Achse durch $\underline{x}$

$X = \mathbb{R}^2 \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} :\Longleftrightarrow \underbrace{\sqrt{x_1^2 + x_2^2}}_{\text{Länge von } \binom{x_1}{x_2}} = \sqrt{y_1^2 + y_2^2}$

$\left[\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right] = \left\{ \text{Vektoren, die dieselbe Länge wie } \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \text{ haben} \right\}$



$$\begin{array}{ccc} \parallel & & \parallel \\ X & \longrightarrow & X/\sim \end{array}$$

enthält ein Element pro möglichen Radius

$X = \mathbb{R}^2 \quad \binom{x_1}{x_2} \sim_f \binom{y_1}{y_2} :\iff (x_1 < x_2$ und $y_1 < y_2)$

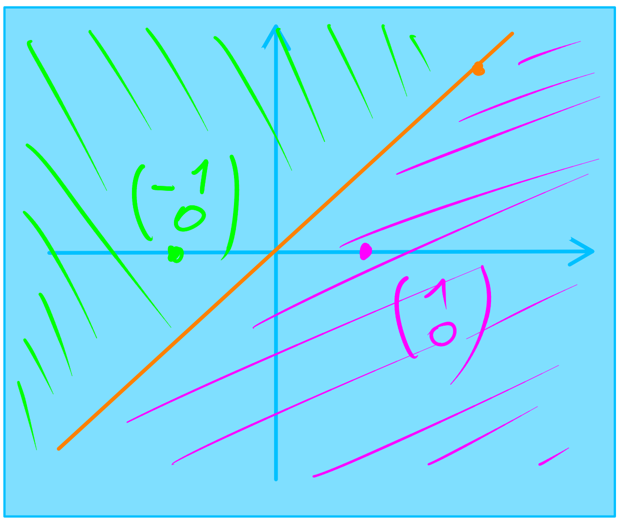
oder

$(x_1 = x_2$ und $y_1 = y_2)$

oder

$(x_1 > x_2$ und $y_1 > y_2)$

$x = \binom{5}{5}$

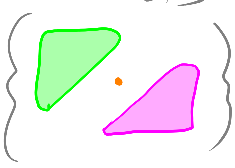


$$\binom{x_1}{x_2} \longmapsto \begin{cases} < & \text{falls } x_1 < x_2 \\ = & \text{"} \quad x_1 = x_2 \\ > & \text{"} \quad x_1 > x_2 \end{cases}$$

$$X \xrightarrow{\quad f \quad} \{>, =, <\}$$

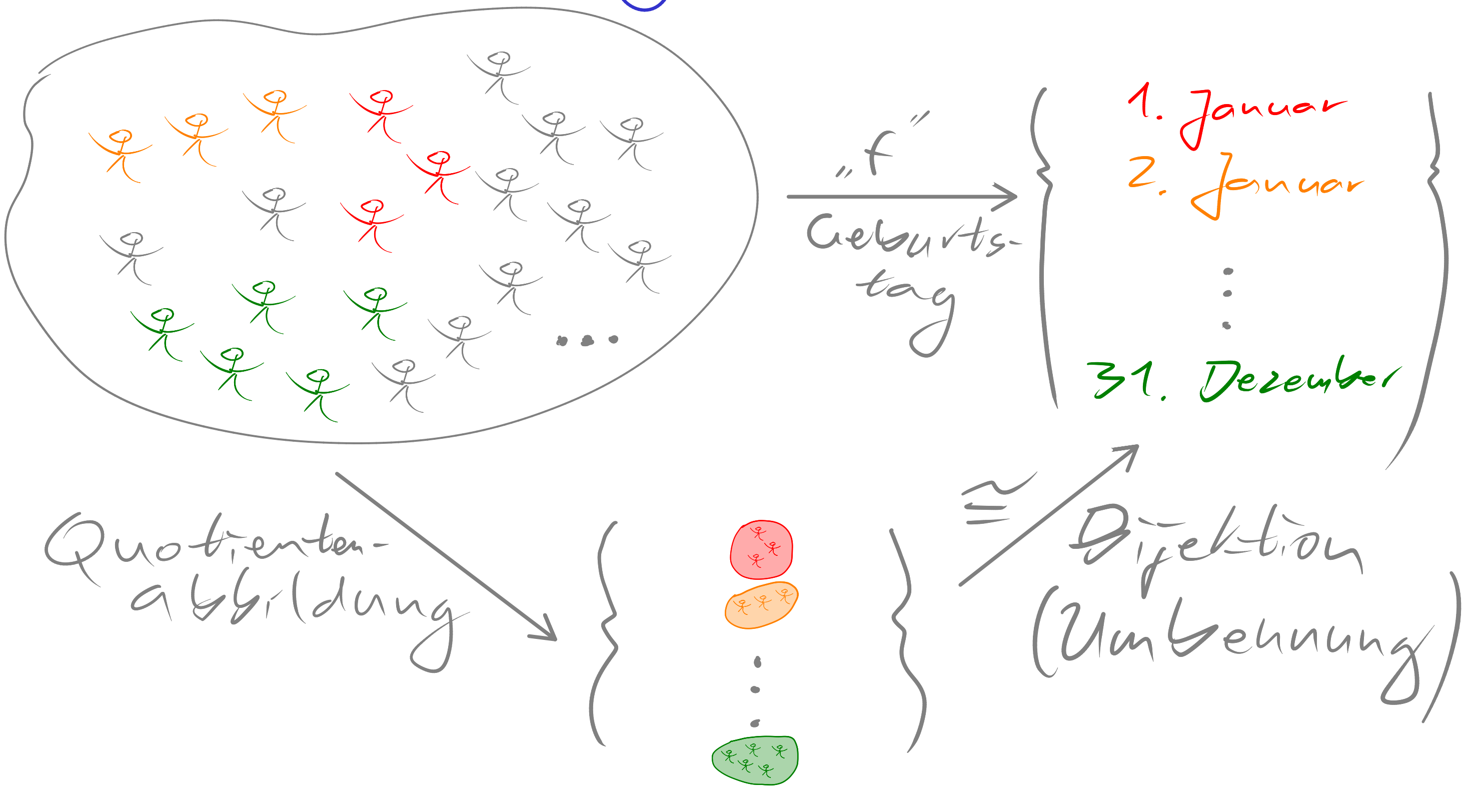
Quotientenabbildung

$[\binom{x_1}{x_2}]$



$\cong$ (Umbenennung)

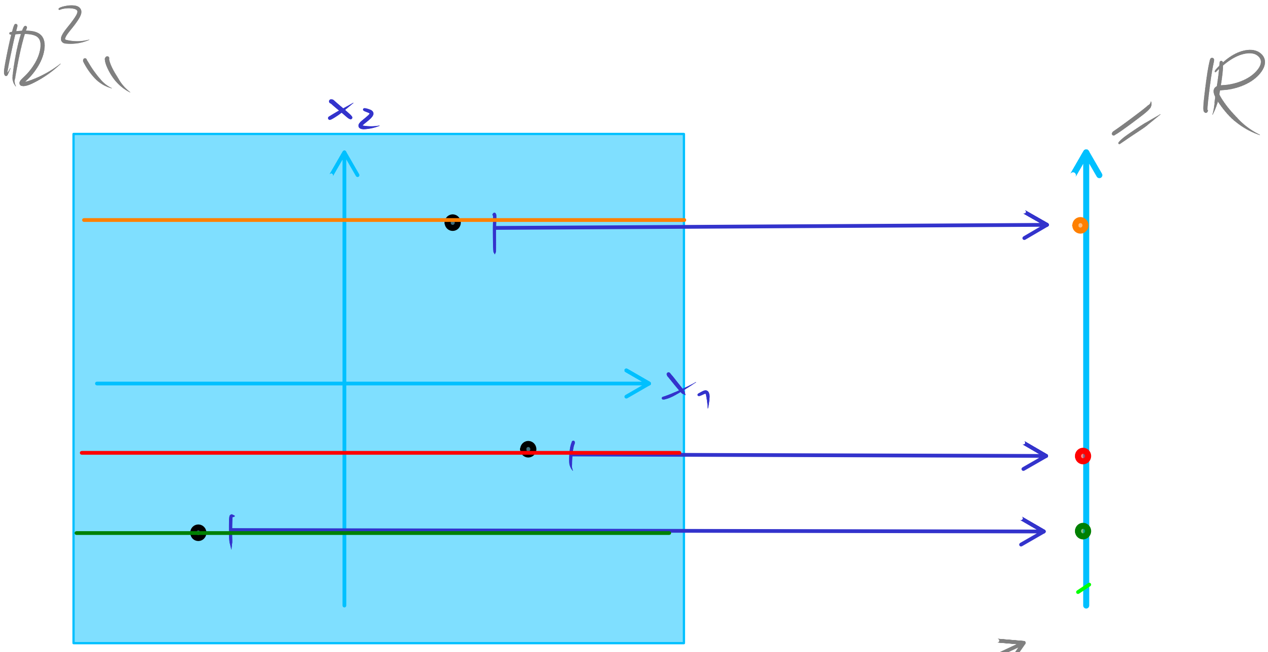
$X =$ Weltbevölkerung



$x \sim_{\text{Geburtstag}} y \iff x$ und y haben denselben Geburtstag

$[x] =$ Faser(Geburtstag von x)

$$\begin{aligned} \mathbb{R}^2 &\xrightarrow{\pi_2} \mathbb{R} \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &\longmapsto x_2 \end{aligned}$$



$$\begin{tikzcd} \mathbb{R}^2 \arrow[dr] & & \mathbb{R} \\ & \mathbb{R}^2/\sim \arrow[ur, "\cong"] & \end{tikzcd}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longmapsto \left\{ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R}^2 \;\middle|\; y_2 = x_2 \right\} \longmapsto x_2$$

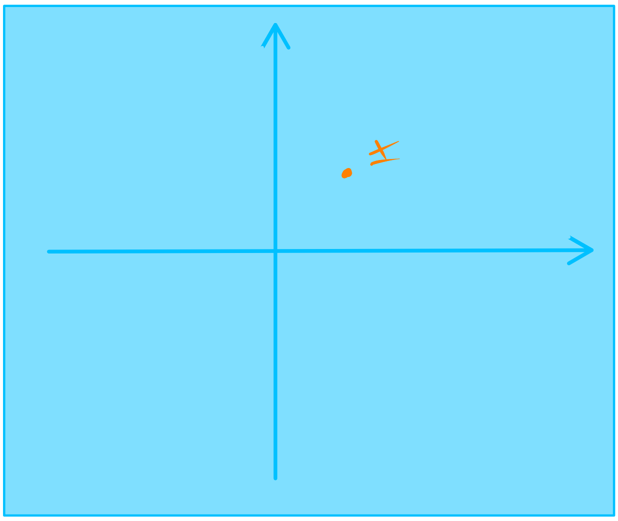
$$x \sim_{\pi_2} y \iff x_2 = y_2$$

$[x] =$ Parallele zur x_1-Achse durch x

$$= \pi_2^{-1}(x_2) = \pi_2^{-1}(\pi_2(x))$$

$$\begin{array}{ccc} \mathbb{R}^2 & \xrightarrow{r} & \mathbb{R}_{\geq 0} \\ \underline{x} & \longmapsto & \sqrt{x_1^2 + x_2^2} \end{array}$$

$\mathbb{R}^2$

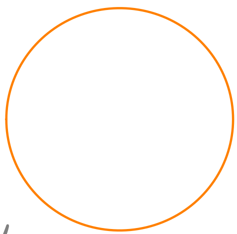


$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "r"] \arrow[d, "\text{Quotienten-abb.}"'] & \mathbb{R}_{\geq 0} \\ \mathbb{R}^2/\sim_r \arrow[ur, "\cong"] & \end{tikzcd}$$

Länge von $\underline{x}$

$\sqrt{x_1^2+x_2^2}$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto$ Kreis durch $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$



Für $\ell \in \mathbb{R}_{\geq 0}$ ist

$$r^{-1}(\ell) = \{ \underline{x} \mid r(\underline{x}) = \ell \}$$

$$[\underline{x}] = \{ \underline{y} \mid r(\underline{y}) = r(\underline{x}) \}$$

$$\sim_r \quad = r^{-1}(r(\underline{x}))$$

$$X = \mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$$

$$\begin{tikzcd}
\mathbb{Z} \times (\mathbb{Z} \setminus \{0\}) \arrow[rr, "f"] \arrow[dr] & & \mathbb{Q} \\
& \left(\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})\right)/\sim \arrow[ur, "\cong"] &
\end{tikzcd}$$

$$(x,y) \longmapsto \frac{x}{y}$$

$$(x,y) \sim_f (a,b) \iff \frac{x}{y} = \frac{a}{b}$$

$$(x,y) \sim (a,b) \iff xb = ya \in \mathbb{Z}$$