EXERCISE SHEET 1

ADVANCED SEMINAR ON GROUP THEORY
SUMMER 2019

PROJECTIVE REPRESENTATION THEORY

All problems are taken out of Isaacs book [1]. If there is interest, we can meet to discuss the problems on Monday.

Problem 1: For a group G and a character χ of G define

$$\mathrm{Z}(\chi) = \{g \in G \mid |\chi(g)| = \chi(1)\}.$$

Using the fact that $\chi(1)|[G : \mathrm{Z}(\chi)]$, proof the last theorem of Tuesdays talk:
(6.15, Ito) Let $A \trianglelefteq G$ be an abelian normal subgroup. Then $\chi(1)|[G : A]$.

Problem 2: Let $N \trianglelefteq G$ and $\varphi, \theta \in \mathrm{Irr}(N)$ be invariant under G and such that $\varphi\theta$ is again irreducible. Assume that θ extends to $\chi \in \mathrm{Irr}(G)$, i.e. $(\chi)_N = \theta$. Define

$$\mathcal{S} = \{\beta \in \mathrm{Irr}(G) \mid [\varphi^G, \beta] \neq 0\} \text{ and } \mathcal{T} = \{\psi \in \mathrm{Irr}(G) \mid [(\varphi\theta)^G, \psi] \neq 0\}.$$

Show that $\beta \mapsto \beta\chi$ defines a bijection $\mathcal{S} \to \mathcal{T}$.

Problem 3: Let $N \trianglelefteq G$ such that G/N is abelian. Let C be the group of linear characters of G/N, acting on $\mathrm{Irr}(G)$. Let $\theta \in \mathrm{Irr}(N)$. Show that:

$$\theta^G = f \sum_{i=1}^{s} \chi_i,$$

where f, s are integers and the set of the$\chi_i \in \mathrm{Irr}(G)$ is an orbit under C.

Problem 4: Let $N \trianglelefteq G$ be such that G/N is cyclic. Let $\theta \in \mathrm{Irr}(G)$ be invariant under G and $\theta(1)$ be coprime to $[G : N]$. Show that θ extends to G.

References

[1] I. M. Isaacs, *Character theory of finite groups*. Pure and Applied Mathematics, No. 69. Academic Press [Harcourt Brace Jovanovich, Publishers], New York-London, 1976.