Oberseminar SoSe 2019: Knots and Primes

Talk 1: Basic knot theory and the knot group *(5.4.; Benjamin Klopsch)*

Introduce basic knot theory including the knot group, following [Mor, Ex. 2.6] (which ends on p. 16) or [Li, Sect. 2]. If necessary, also quickly recall some of the material of [Mor, Sect. 2] before Ex. 2.6.

Talk 2: Coverings and knots *(12.4.; Benno Kuckuck)*

Recall the basics of universal coverings, in particular [Mor: Prop 2.7, Thm 2.8], monodromy permutation representation [Li, Prop. 6.1]; explain the infinite cyclic covering and Seifert surfaces [Mor, Ex. 2.12] / [Li, Ex. 4.8, 4.9]; introduce the abelian fundamental group, maximal abelian covering [Mor, Ex. 2.13]. Introduce the ramified coverings from [Mor, Ex. 2.14], [Li, Sect. 4]

Talk 3: Finite etale coverings *(26.4.; Johannes Fischer)*

Recall quickly how rings give rise to affine scheme; introduce finite etale maps, as analogon to finite unramified coverings; give many examples; explain the analogy to the topological world

Literature: [Mor, Sect. 2.2 up to middle of p. 28], [Mor, Ex. 2.17–2.19]; see also [Li, Sect. 5]

Talk 4: The etale fundamental group *(3.5.; Leif Zimmermann)*

Quickly recall what the pro-finite completion of a group is; introduce the etale fundamental group (giving the analogy to the usual fundamental group and its relation to coverings); give many examples

Literature: [Li, Sect. 8, up to Ex. 8.10], [Li, Ex. 8.5], [Mor, Thms 2.23, 2.24], [Mor, Ex. 2.25, 2.26]; see also [Li, Sect. 5]

Talk 5: Number rings *(10.5.; Hamed Khalilian)*

Recall some terminology and facts about number rings, following [Mor], from the middle of p. 33 on. In particular recall [Mor, Thms 2.29 - 2.31]; introduce the notion of ramification for number field extensions, as an analogue to topogical ramification; determine the etale fundamendal groups of $\mathrm{spec}(\mathbb{Z}_p)$ (the p-adic integers) and $\mathrm{spec}(\mathbb{Z})$ [Mor, Ex 2.34, 2.35]. If time permits, also say something about [Mor, Ex 2.33].

Talk 6: The prime group *(17.5.; Benedikt Schilson)*

Introduce the prime group, as an analogue to the knot group (see [Mor, p. 51], [Li, Sect. 9]), and provide some understanding of it. More precisely, present [Mor, Ex 2.36] (maybe only for $k = \mathbb{Q}$), introduce the group $G_S(k)$ (maybe only for $k = \mathbb{Q}$). Also present [Mor, Ex 2.40] and if time permits, determine $G_{(q)}(\mathbb{Q})^{ab}$ for primes q, via [Li, Thm 9.8].

Talk 7: Introduction to class field theory *(24.5.; Matteo Vannacci)*

Give a quick introduction to class field theory as in [Mor, beginning of Sect. 2.3]. In particular, explain why locally constant, finite etale sheaves are simply $\pi_1(X, \bar{x})$-modules. Introduce the etale cohomological dimension and compute it for finite fields [Mor, Sect. 2.3.1]. In particular, recall what all the things mean in the middle line of [Mor, Sect. 2.3.1] (about the cup-product). (I did not yet search for a good reference for that; ask me if you need one.)

Talk 8: Local class field theory *(31.5.; Kevin Langlois)*

Present [Mor, Sect. 2.3.2]; in particular, explain the local Tate duality [Mor, 2.41] (and why local fields have tale cohomological dimension 2) and the Hilbert symbol

Talk 9: $\mathrm{spec}\,\mathbb{Z}$ **is 3-dimensional** *(7.6.; Thuong Dang)*

The main goal of this talk is to make sense of $\mathrm{spec}\,\mathbb{Z}$ being 3-dimensional (see [Mor, middle of p. 50]). The key is Artin-Verdier-duality [Mor, 2.42]. Try to give an intuition of what is happening without going too much into technical details. Maybe present [Mor, Ex. 2.44]. If time permits, also explain why/how the Tate-Poitou sequence [Mor, 2.43] corresponds to the relative cohomology sequence, as claimed at the end of [Mor, Sect. 2.3.3].

Talk 10: Guest talk: Vector fields and moduli of canonically polarized surfaces in positive characteristics *(14.6.; Nikolaos Tziolas; Cyprus)*

Talk 11: Overview over the analogy between primes and knots *(21.6.; Marcus Zibrowius)*

This talk does not contain so much new mathematics; instead, its goal is to give an overview over the analogy between knots and primes, building on what we already know, as in [Mor, Sect. 3]; see also [Li, p. 3]. In particular, explain how local fields come into play. Also explain what the prime group is and why it is the analogon of the knot group. See also [Li, Sect. 9]

Talk 12: Decomposition of knots and primes *(28.6.; Stefan Schrer)*

Present [Mor, Sect. 5.1], which describes how knots decompose when one lifts them using a Galois covering, and [Mor, Sect. 5.2], which describes how primes decompos in Galois extensions. Explain the connection between decomposition of knots and primes.

Talk 13: Linking number and Legendre Symbols *(5.7.; Moritz Petschick)*

Explain in which sense Linking numbers correspond to Legendre Symbols [Mor, Sect. 4] and how quadratic reciprocity is related to the symmetry of linking numbers. If time permits, also explain the comment at the end of [Mor, Sect. 4] which says that the integral formula for linking numbers corresponds to the Gauss sum for the Legendre symbol.

See also [Li, Sect. 7]

Literature

[Mor] Morishita: Knots and Primes. `https://link.springer.com/book/10.1007%2F978-1-4471-2158-9`

[Li] Li: Knots and Primes. (attached)