L.18.6

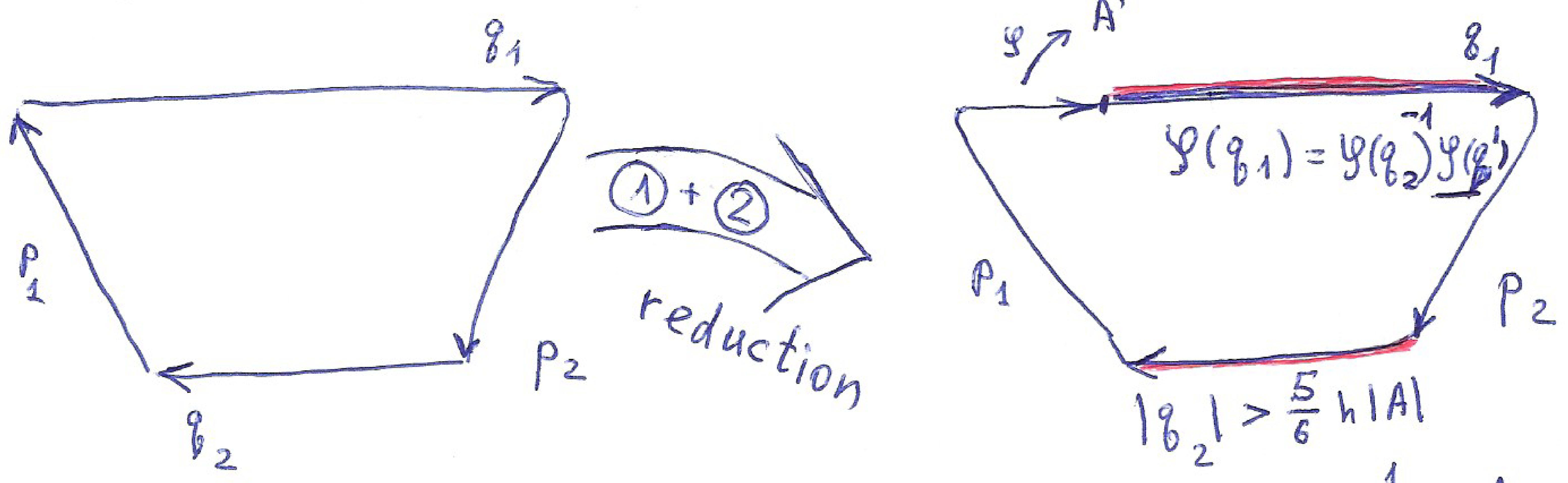
Induction by $L = |A| + |A|$ prove that q_1, q_2 are A-compatible.

$h = \delta^{-1}$

$r(\Delta) = i$, A = period simple in r.i.

$\varphi(q_1)$, $\varphi(q_2)^{-1}$ are A-periodic

$|A'| < \frac{1}{2}|A|$ $\quad |q_1| > \frac{5}{6}h|A|$



$|p_1|, |p_2| < \alpha|A|$

$|q_1|, |q_2| > (\frac{5}{6}h + 1)|A|$

$|p_1| < \bar{\alpha}|A|$ $\quad \varphi(q_2^{-1}) = A\ldots$

Thm 17.1

smooth L.19.4-5

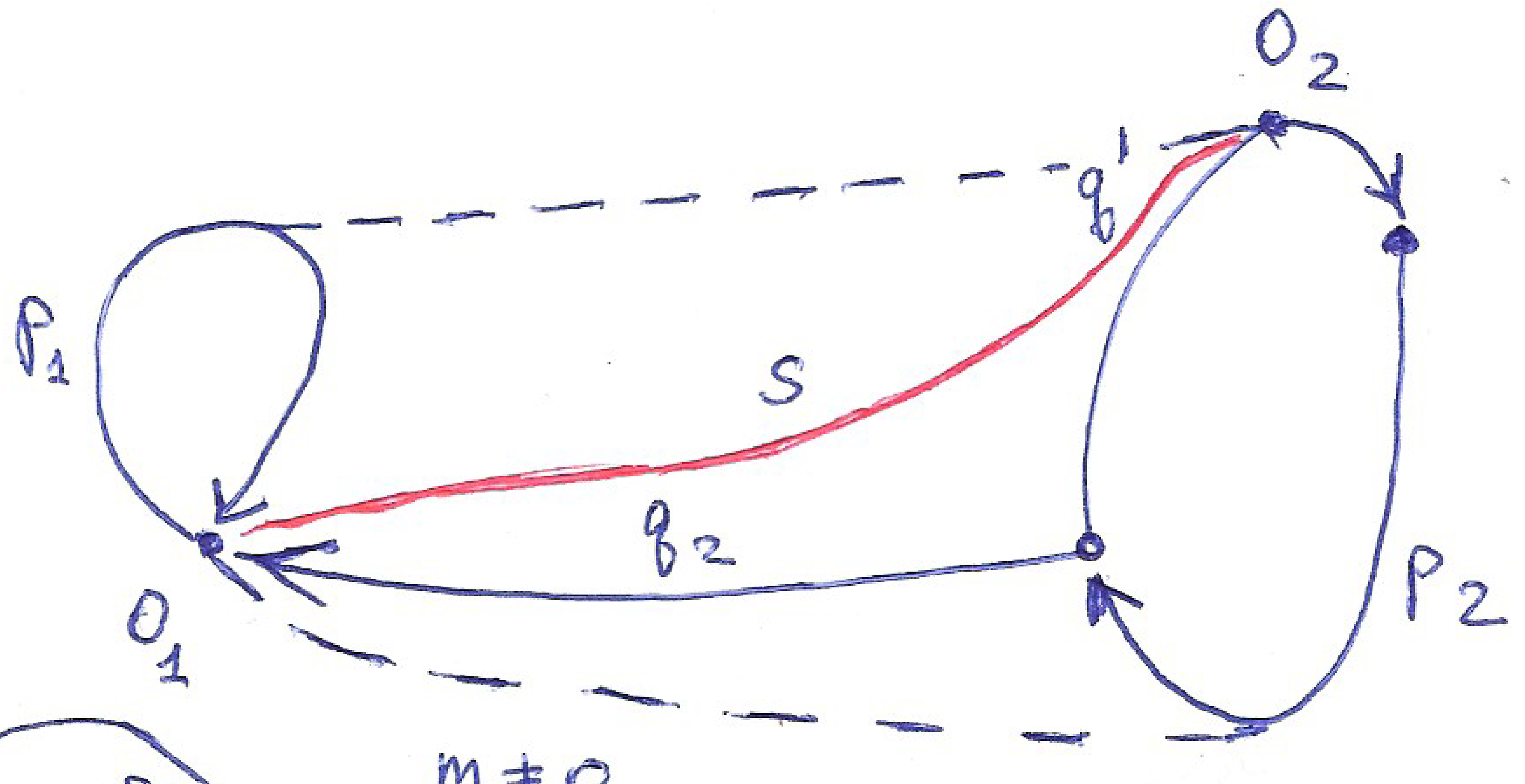
$\bar{\beta}|q_1| < |q_2| + |p_1| + |p_2| <$

$|q_2| + (\bar{\alpha} + \alpha)|A|$

$\Downarrow$

$|q'| = |q_1| - |q_2| < (\bar{\beta}^{-1} - 1)|q_2| + |A|$

$\varphi(s)$ is A-periodic, $|s| \geq |q_2|$, $\bar{s}$ and s are homotopic



$m \neq 0$

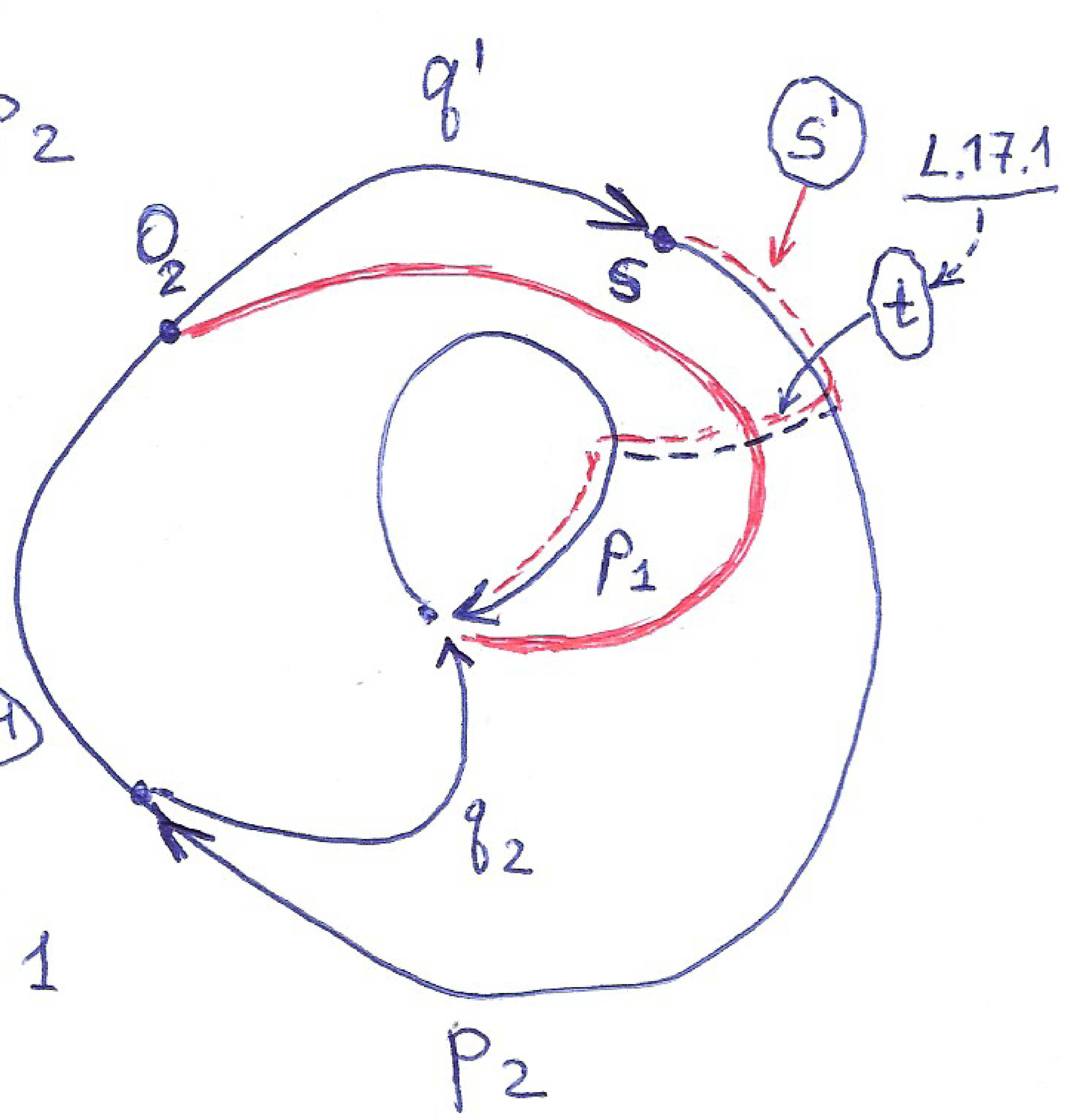
simple in r.i or a period of r.k.s i

$Y B^m Y^{-1} \overset{i}{=} \varphi(p_1) = D$

$*i$

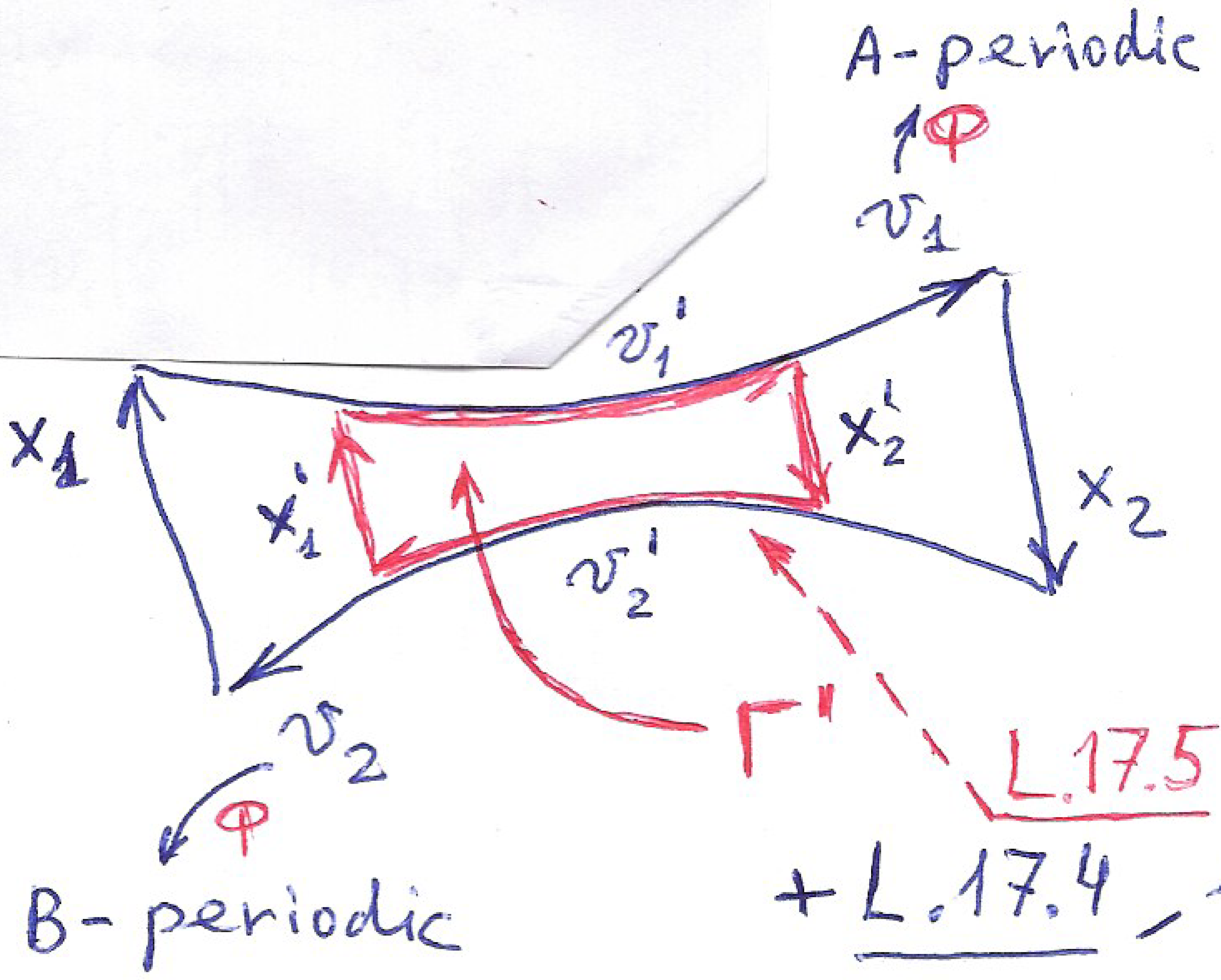
1

$\varphi(\bar{s}) \overset{i}{=} D^l \varphi(s')$ ← L.11.4



Let $Z = Y^{-1}\varphi(s')$

Then $\underset{x_1}{Y^{-1}}\ \underset{v_1}{\varphi(s)}\ \underset{x_2}{Z^{-1}}\ \underset{v_2}{B^{-ml}} \overset{i}{=} 1$



L.17.5

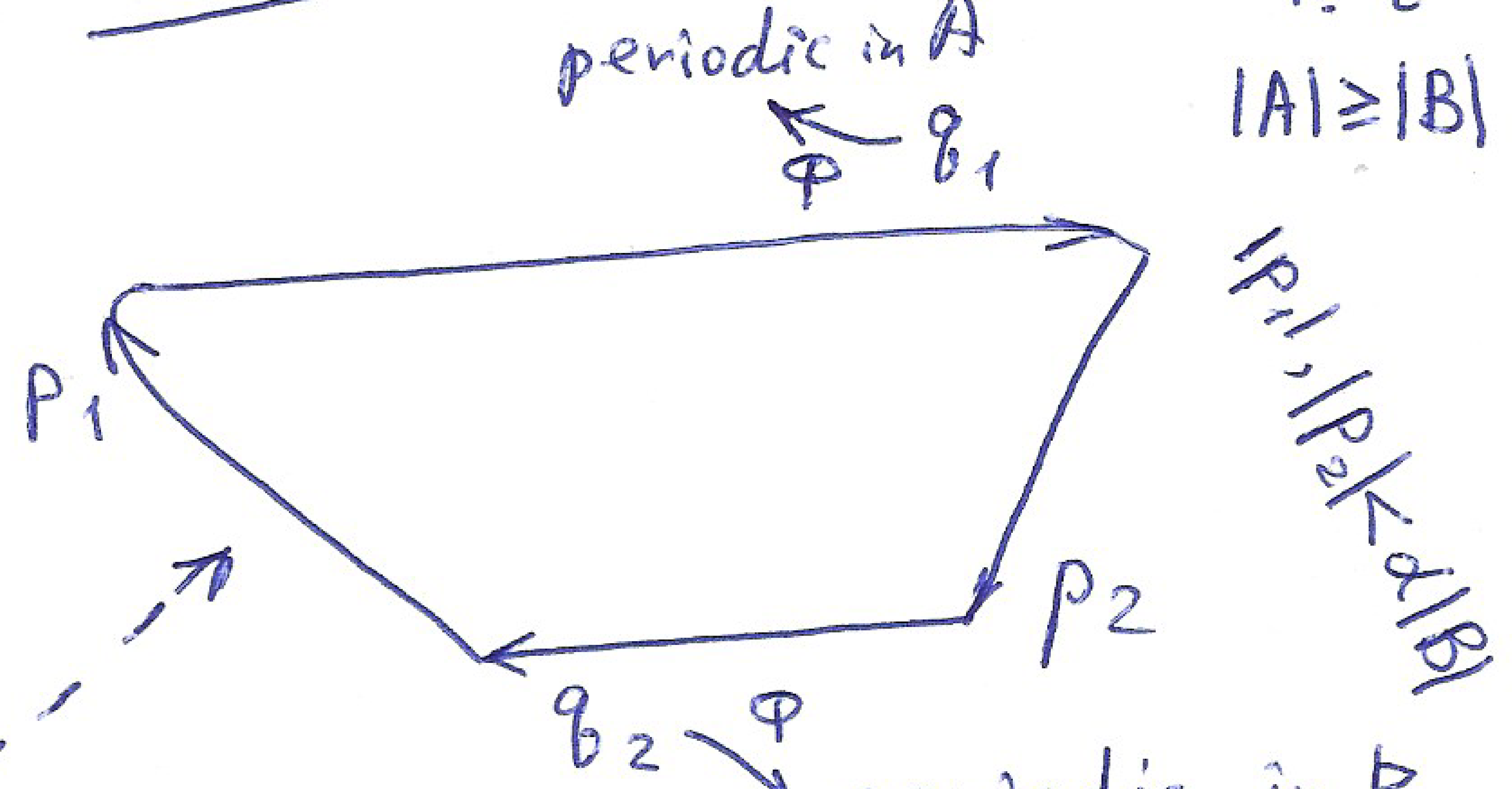
+ L.17.4

$j = r(\Gamma') < |B| = k$

L.18.8

A, B = simple periods in r. i

$|A| \geq |B|$



$|p_1|, |p_2| < \alpha |B|$

$|q_2| > h|B|$

$|q_1| > \frac{3}{4} h |A|$

- Then $A \overset{i}{\sim} B^{\pm 1}$

impossible in Γ' because $|B| < |A|$ and A is simple in r. i

- Similar argument for L.18.7:

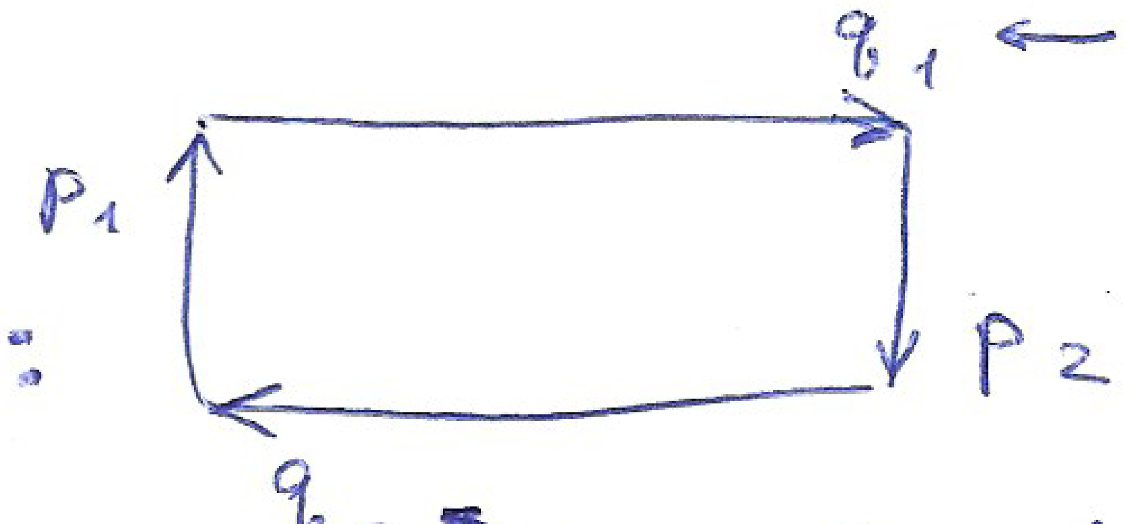
$\varphi(x_l) = Z_l$, $\varphi(v_l) = A^{\pm m_l}$ simple in r. i

$|Z_1| + |Z_2| < (\gamma(\min(m_1, m_2) - \frac{5}{6}h - 1) - 1)|A|$

- Then $Z_1 \overset{i}{=}$ power of $A \overset{i}{=} Z_2$ (using L.18.6)

L.17.4

A-map Δ:



q_1 ← smooth r. $k \leq l$

q_2 ← smooth r. l

$|p_1|, |p_2| < \zeta n k$

- Then $r(\Delta) < 3\gamma^{-1}\zeta k$ $\hat{k}$

- Then $|\partial \Pi| < 3\gamma^{-1}\zeta n k < nk$

L.17.5 When $M \leq |q_i|$, then $\exists\ p_1' q_1' p_2' q_2'$: $|q_j'| > |q_j| - M$

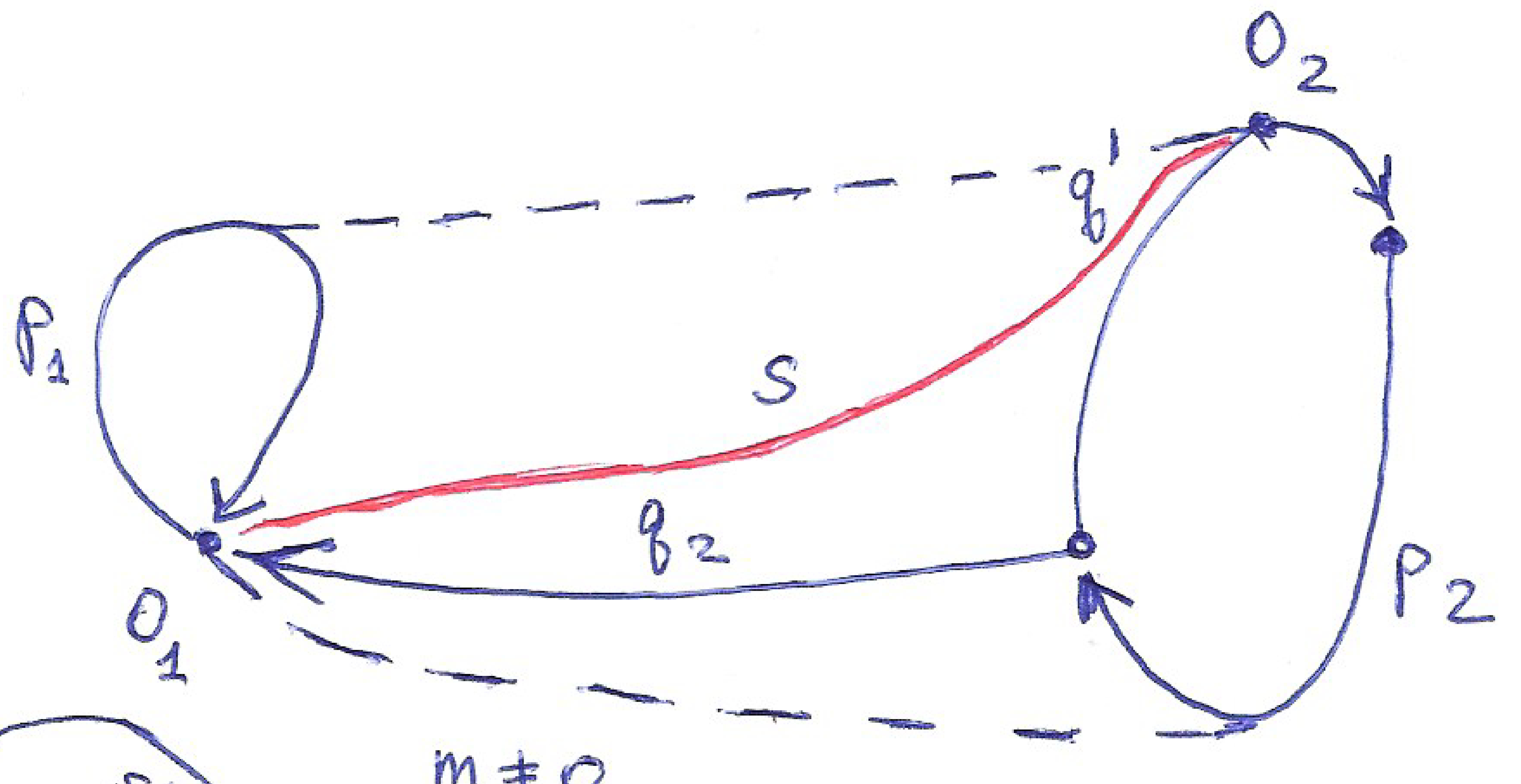
$M = \gamma^{-1}(|p_1| + |p_2| + k)$

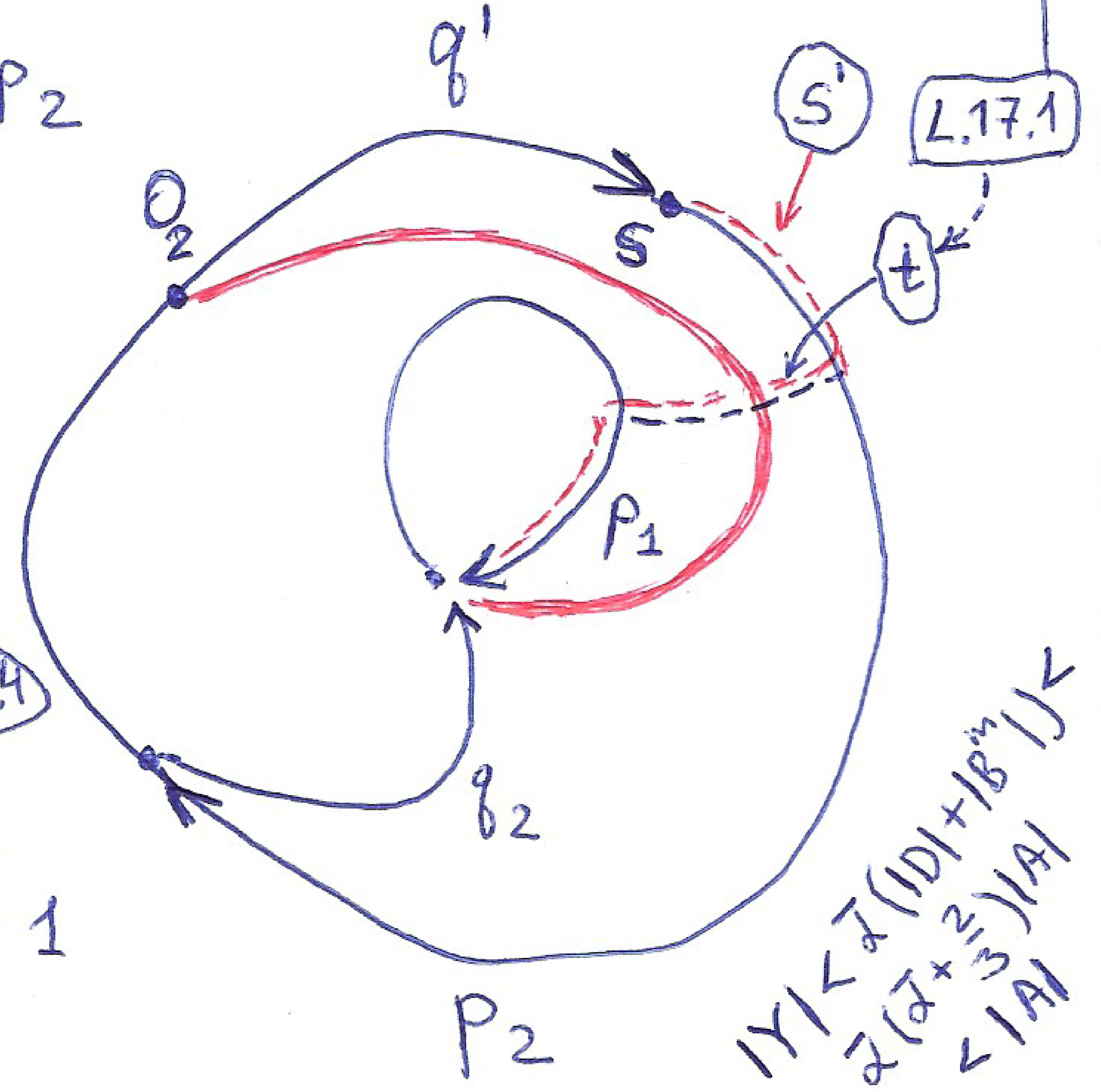
$|p_j'| < \alpha k$

$\varphi(s)$ is A-periodic , $|s| \geq |q_2|$, $\bar{s}$ and s are homotopic

$|t| < \gamma(|p_1|+|p_2|+|q'|) < |A| + \gamma(\bar{\beta}^{-1}-1)|q_2|$

$|s'| < |t| + \frac{1}{2}(|p_1|+|p_2|) < |t| + |A|$





$m \neq 0$

simple in r. i or a period of r. $k \leq i$

$$YB^mY^{-1} \overset{i}{=} \varphi(p_1) = D \overset{*i}{\neq} 1$$

$$\varphi(\bar{s}) \overset{i}{=} D^{\ell}\,\varphi(s') \quad \text{L.11.4}$$

Let $Z = Y^{-1}\varphi(s')$

Then $\underset{x_1}{Y^{-1}}\;\underset{v_1}{\varphi(s)}\;\underset{x_2}{Z^{-1}}\;\underset{v_2}{B^{-m\ell}} \overset{i}{=} 1$

$|B| \leq |B^m| < \bar{\beta}^{-1}|D| < \bar{\beta}^{-1}\alpha|A| < \frac{2}{3}|A|$

$|Y| < \alpha(|D|+|B^m|) < \alpha(\alpha+\frac{2}{3})|A| < |A|$

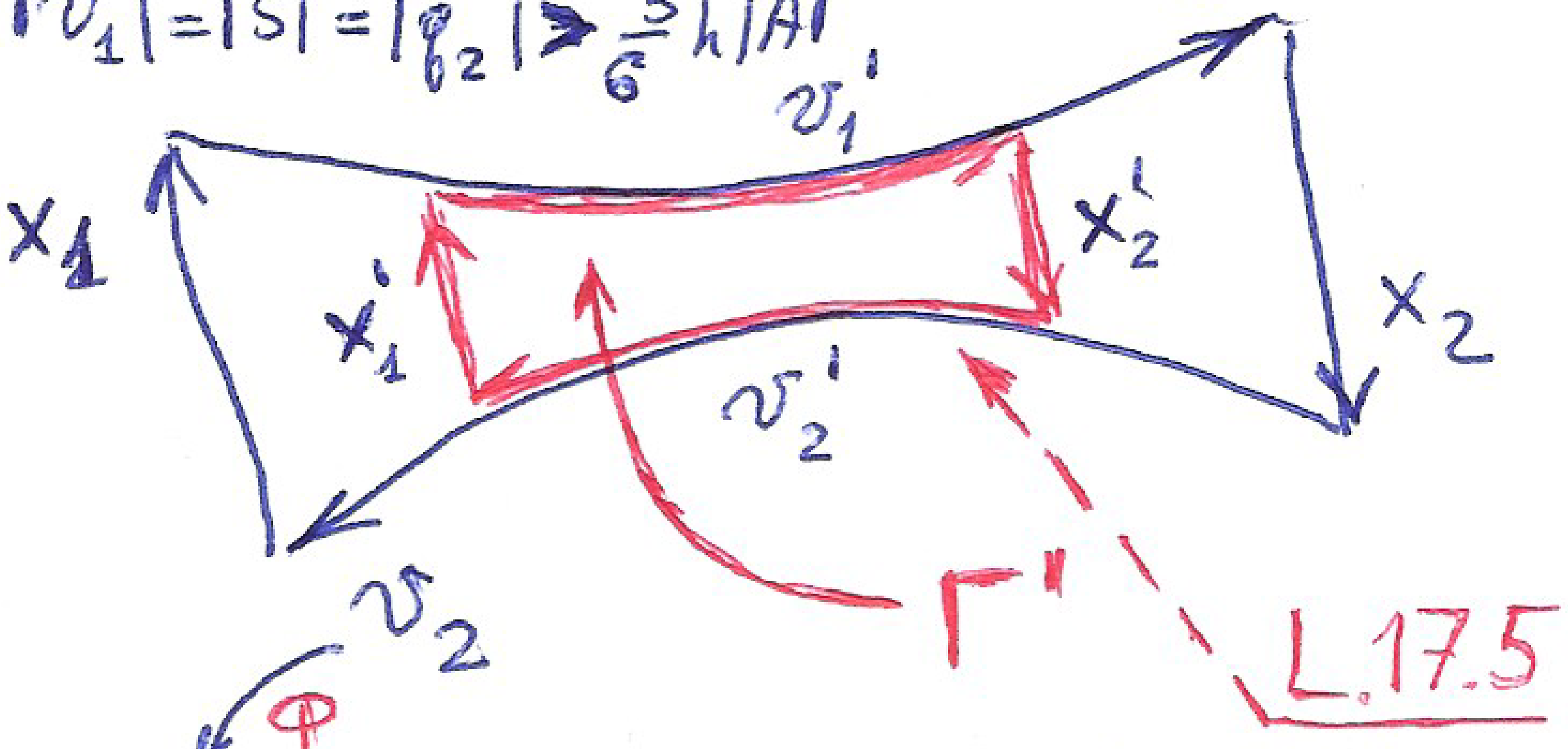
$|x_1| < |A|$

$|x_2| < \gamma(\bar{\beta}^{-1}-1)|q_2| + 3|A|$

$|v_1| = |s| = |q_2| \geq \frac{5}{6}h|A|$

A-periodic

$\uparrow \varphi$



$\swarrow \varphi$

B-periodic

+ L.17.4

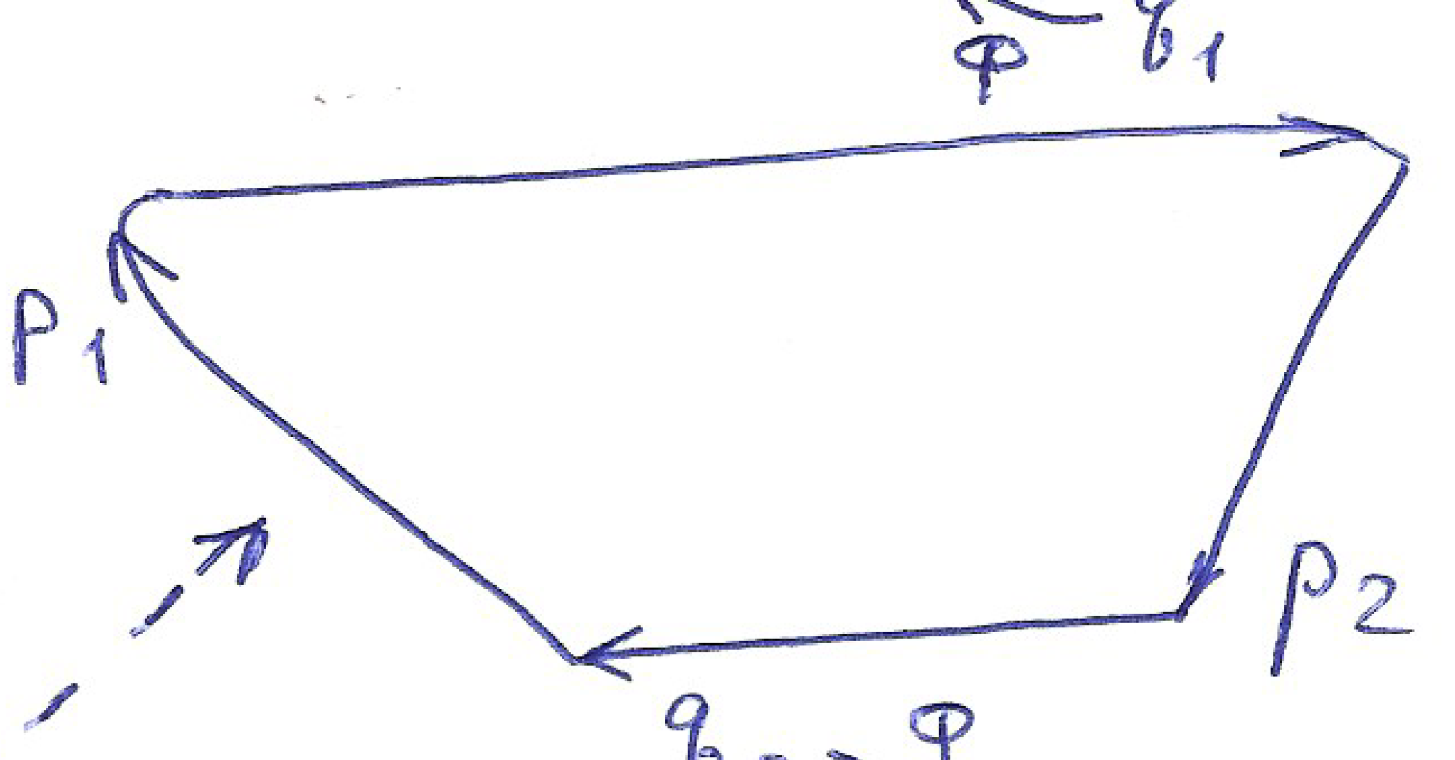
$j = r(\Gamma') < |B| = k$

L.18.8 A, B = simple periods in r. i

$|A| \geq |B|$

periodic in A

$\overset{\varphi}{\longleftarrow} q_1$



$|p_1|, |p_2| < \alpha|B|$

$q_2 \overset{\varphi}{\longrightarrow}$ periodic in B

$|q_2| > h|B|$

$|q_1| > \frac{3}{4}h|A|$

• Then $A \overset{i}{\sim} B^{\pm 1}$

impossible in Γ' because $|B| < |A|$ and A is simple in r. i

• Similar argument for L.18.7:

$\varphi(x_\ell) = Z_\ell$, $\varphi(v_\ell) = A^{\pm m_\ell}$ simple in r. i

$|Z_1|+|Z_2| < (\gamma(\min(m_1,m_2) - \frac{5}{6}h - 1) - 1)|A|$

• Then $Z_1 \overset{i}{=}$ power of $A \overset{i}{=} Z_2$ (using L.18.6)