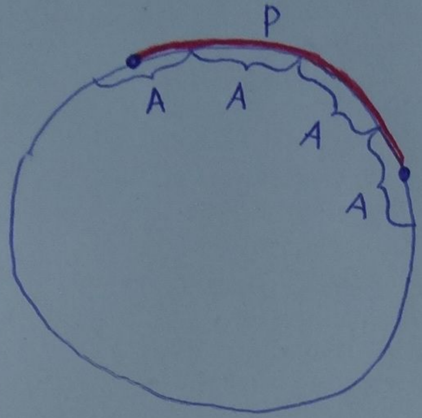
(1)

Lem 19.5. Let Δ be reduced of rank i

Let p be a section of $\partial\Delta$ whose label $\varphi(p)$ is A-periodic, where



- A is simple in rank i:

$$A \underset{G(i)}{\not\sim} B^k, \quad k=1,\dots,n \qquad (*)$$

$$A \underset{G(i)}{\not\sim} A', \quad |A'|<|A|$$

or
- A is a period of rank $j \le i$

  (i.e. $|A|=j$ and $(*)$)

  and Δ has no cells of rank j A-compat. with p.

Additionally: If p is a cyclic section, then we require $\varphi(p)=A^m$ for some m.

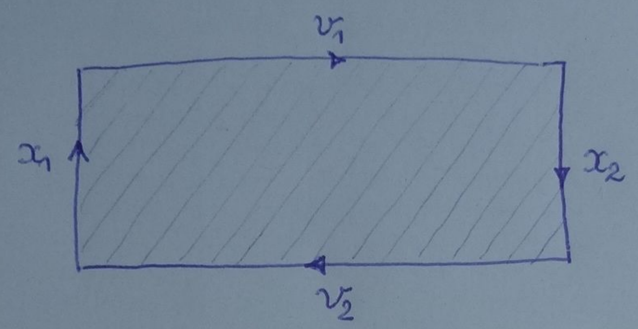
Then p is $|A|$-smooth.

②

Part 4) of the proof of Lemma 18.6

$$Y^{-1}\varphi(s)Z^{-1}B^{-ml} \underset{G(i)}{=} 1$$

Γ:



$\boxed{\text{Idea}}$ $|x_1|, |x_2|$ are small, $|v_1|, |v_2|$ are ~~large~~ smooth with many A and B-periods $\overset{18.8}{\Longrightarrow}$ $A \underset{G(i)}{\sim} B^{\pm 1}$, a contr. since A is simple rk i and $|B| < |A|$ by (5).

$$|x_1| = |Y| \underset{(6)}{<} |A|$$

from formulation L. 18.6

$$|x_2| = |Z| = |Y^{-1}\varphi(s')| \underset{(6)+(4)}{<} |A| + (\gamma(\beta^{-1}-1)|q_2| + 2|A|)$$

$$|v_1| = |S| \geq |q_2| \underset{(2)}{>} \frac{5}{6} h |A|$$

earlier

(h is large since $h^{-1} = \delta < \gamma$)

$$\varphi(v_2) = B^{-ml}$$

By Lemma 19.5, v_1 is $|A|$-smooth, v_2 is $|B|$-smooth.

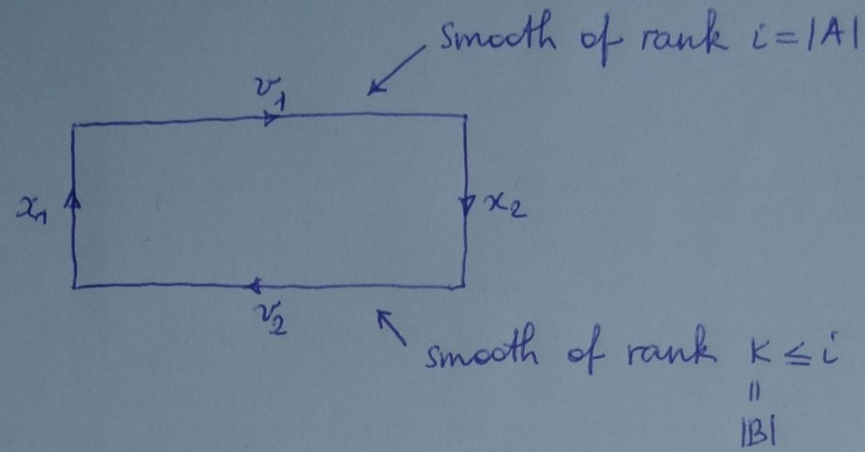
<u>Indeed</u>: v_1 has label $\varphi(s)$ ← was A-periodic with A-simple in rk $= i$.

v_2 has label B^{-ml} ← B is simple in rk i or a period of rk $k \leq i$

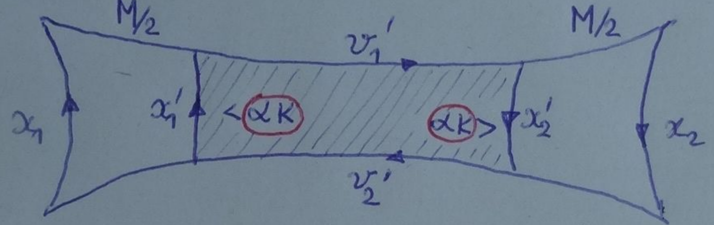
was deduced from Lem. 18.1

$\Rightarrow$ <u>Note</u>: B is simple in rank $k \leq i = |A|$.

③



We want to apply Lem. 17.5



$M = \gamma^{-1}(|x_1| + |x_2| + K)$

We shall verify the conditions of Lem 17.5. $|v_1| > M$ and $|v_2| > M$.

Ex.: $|v_1'| = |v_1| - \gamma^{-1}(|x_1| + |x_2| + \overset{K}{\cancel{|A|}})$

$> |q_2| - \gamma^{-1}\left(\gamma(\beta^{-1}-1)|q_2| + 5|A|\right)$ (see page 2)

$= (2-\beta^{-1})|q_2| - 5\gamma^{-1}|A|$

$> (2-\beta^{-1})\cdot\frac{5}{6}h|A| - 5\gamma^{-1}|A|$

$= \left[(2-\beta^{-1})\frac{5}{6}h - 5\gamma^{-1}\right]\cdot|A|$ (Recall $h^{-1} = 5 < \gamma \Rightarrow h > \gamma^{-1}$)

$> \frac{3}{4}h\cdot|A|. > 0.$

No. apply Theorem 17.1 about smooth sections:

$|v_2'| > \beta(|v_1'|) - |x_1'| - |x_2'| > \beta\cdot\frac{3}{4}h|A| - 2\alpha\cdot|A| \gtrsim \frac{2}{3}h|A| \overset{(5)}{>} h\cdot|B|.$

Lem. 18.8 says that in this situation $A \underset{G(i)}{\sim} B^{\pm 1}$ (we can apply it since $|A|+|B|<|A|+|A|$)

Lem 18.8 says that in this situation $A \underset{G(i)}{\sim} B^{\pm 1}$

(Note, we can apply it since $|A| + |B| < |A| + |A|$, induction.)

But A is simple in rank i and $|B| \underset{(5)}{<} \frac{2}{3}|A|$.

A contradiction.

$$\Downarrow$$

The assumption that $\varphi(p_1) \underset{G(i)}{\neq} 1$ is wrong (see page 200)

$\Rightarrow$ q_1 and q_2 are A-compatible in Δ.