# Cryptography

## Series 1

**Problem 1.**                                                   [4+6 points]

(a) Find the greatest common divisor $\gcd(78, 34)$.

(b) Find some integer numbers $u, v$ such that $78u + 34v = \gcd(78, 34)$.

**Problem 2.**                                                   [4+6 points]

(a) Let $a$ and $n$ be two coprime natural numbers. Prove that there exists a natural number $u$ such that
$$au \equiv 1 \ (\bmod \ n).$$

(Hint: Use Theorem 1.1 about Euclidean algorithm.)

(b) Find all integer numbers $x$ such that

$$\begin{cases} x \equiv 1 \ (\bmod \ 3), \\ x \equiv 2 \ (\bmod \ 5), \\ x \equiv 3 \ (\bmod \ 8). \end{cases}$$

(Hint: Use Chinese remainder theorem.)

**Problem 3.** Let $F_n$ be the $n$-th Fibonacci number.          [4+6 points]

(a) Find the minimal prime number $n > 2$ such that $F_n$ is not a prime number.

(b) Prove Cassini's identity $F_{n-1}F_{n+1} - F_n^2 = (-1)^n$.