Mathematisches Institut
Heinrich-Heine-Universität Düsseldorf
Prof. Dr. Stefan Schröer

WS 21/22

Algebraic Geometry I

Sheet 6

Excercise 1. Let R be a ring and $n \geq 1$ be some integer. We consider the polynomial ring $S = R[u, v]$ as graded with respect to the monoid $\Lambda = \mathbb{Z}/n\mathbb{Z}$ of congruence classes modulo n, by declaring

$$\deg(u) = 1 \quad \text{and} \quad \deg(v) = -1.$$

Show that the subring S_0 is isomorphic to $R[x, y, z]/(z^n - xy)$.

Exercise 2. Let $S = \bigoplus_{\lambda \in \Lambda} S_\lambda$ be a Λ-graded ring, and $\mathfrak{a} \subset S$ be an ideal. Verify that the subgroup

$$\mathfrak{a}^{\mathrm{hgs}} = \bigoplus_{\lambda \in \Lambda} (\mathfrak{a} \cap S_\lambda)$$

inside S is a homogeneous ideal contained in $\mathfrak{a}$, and indeed the largest such ideal. Furthermore, check that $\mathfrak{p}^{\mathrm{hgs}}$ is prime if $\mathfrak{p}$ is prime, provided $\Lambda = \mathbb{N}$.

Exercise 3. Let $S = \bigoplus_{i \geq 0} S_i$ be a graded ring. Show that the homogeneous spectrum $\mathrm{Proj}(S)$ is empty if and only if the irrelevant ideal S_+ consists only of nilpotent elements.

Exercise 4. Let $S = \bigoplus_{i \geq 0} S_i$ be a graded ring. Prove that S is noetherian if and only if S_0 is noetherian and the S_0-algebra S is of finite type.

Abgabe: Bis Donnerstag, den 2. Dezember um 23:55 Uhr über ILIAS.