Mathematisches Institut
Heinrich-Heine-Universität Düsseldorf
Prof. Dr. Stefan Schröer

WS 21/22

Algebraic Geometry I

Sheet 3

Excercise 1. Let $f : X \to Y$ and $g : Y \to Z$ be morphisms of schemes. Verify the following implications:

(i) If f and g are locally of finite type, the same holds for the composition $g \circ f$.

(ii) If $g \circ f$ is locally of finite type, then f is locally of finite type.

Also give an example where $g \circ f$ is locally of finite type, but g is not.

Exercise 2. Let $f : X \to Y$ be a continuous map of topological spaces, and $\mathscr{F}$ be an abelian sheaf on X. We define an abelian sheaf $f_*(\mathscr{F})$ on Y by declaring

$$\Gamma(V, f_*(\mathscr{F})) = \Gamma(f^{-1}(V), \mathscr{F}).$$

Make the restriction maps explicit, verify that this indeed gives a presheaf, and check the sheaf axiom.

Exercise 3. Let R be a principal ideal domain and $X = \operatorname{Spec}(R)$ the resulting affine scheme. Show that every open set U is affine.

Exercise 4. Let X be a scheme. Show that the following two conditions are equivalent, by using the sheaf axiom:

(i) There is an affine open covering $X = \bigcup_{\lambda \in L} U_\lambda$ such that the rings of local sections $\Gamma(U_\lambda, \mathscr{O}_X)$ are reduced.

(ii) For every open set $V \subset X$ the ring $\Gamma(V, \mathscr{O}_X)$ is reduced.

Recall that a ring R is *reduced* if $g = 0$ is the only nilpotent element $g \in R$. In other words, the zero ideal is a radical ideal.

Abgabe: Bis Donnerstag, den 11. November um 23:55 Uhr über ILIAS.