Mathematisches Institut
Heinrich-Heine-Universität Düsseldorf
Prof. Dr. Stefan Schröer

WS 21/22

Algebraic Geometry I

Sheet 2

Excercise 1. Let X be a scheme, R be a ring, and $\varphi : R \to \Gamma(X, \mathscr{O}_X)$ be a homomorphism. For $x \in X$ we define $f(x) \in \operatorname{Spec}(R)$ as the point corresponding to the kernel for the composition

$$R \xrightarrow{\varphi} \Gamma(X, \mathscr{O}_X) \xrightarrow{\text{res}} \mathscr{O}_{X,x} \xrightarrow{\text{pr}} \mathscr{O}_{X,x}/\mathfrak{m}_x = \kappa(x).$$

Check that the resulting map $f : X \to \operatorname{Spec}(R)$ is continuous.

Exercise 2. Let X be a scheme. Show that X is quasiseparated if and only if there is an affine open covering $X = \bigcup_{\lambda \in L} W_\lambda$ such that the intersections $W_{\lambda\mu} = W_\lambda \cap W_\mu$ are quasicompact.

Exercise 3. Let $(U, \mathscr{O}_U)$ and $(V, \mathscr{O}_V)$ be two schemes. Suppose we have open sets $U' \subset U$ and $V' \subset V$, together with an isomorphism of schemes

$$(f, \varphi) : (U', \mathscr{O}_{U'}) \longrightarrow (V', \mathscr{O}_{V'}).$$

Endow the set-theoretic union $X = U \cup V$, where the points $x \in U'$ are identified with $f(x) \in V'$, with a canonical scheme structure, by declaring a topology and defining the structure sheaf.

Exercise 4. Let T_0 and T_1 be indeterminates. The *projective line* $X = \mathbb{P}^1_R$ over a ring R is defined by the above gluing construction for $U = \operatorname{Spec} R[T_0]$ and $V = \operatorname{Spec} R[T_1]$, and

$$U' = \operatorname{Spec} R[T_0^{\pm 1}] \quad \text{and} \quad V' = \operatorname{Spec} R[T_1^{\pm 1}]) \quad \text{and} \quad \varphi(T_1) = T_0^{-1}.$$

Prove that the affinization of this scheme is given by

$$(\mathbb{P}^1_R)^{\text{aff}} = \operatorname{Spec}(R),$$

by computing the ring of global sections $\Gamma(X, \mathscr{O}_X)$ with the sheaf axiom.

Abgabe: Bis Donnerstag, den 4. November um 23:55 Uhr über ILIAS.