GRK WORKSHOP: LARGE FIELDS

Introduction

Large Fields were introduced by Pop in [4]. They turn out to be a good class of fields to do a lot of interesting mathematics. In this workshop we will in introduce large fields and give one of the *ample* [1] applications in the form of inverse Galois theory. We will only use the barest minimum of notions from model theory and only in the third talk, where one of the criteria for being large is formulated by being existentially closed in the power series field.

In what follows we give a brief description of the lectures program:

Talk 1: Overview

This first lecture should motivate large fields and give a broad overview over the theory, following the rough structure of [5], though not giving any proofs. The applications in Part II can be mentioned. It could also be of interest to mention the most recent result of the stable field conjecture in the case of large fields [3] which is covered in the model theory seminar this semester.

Talk 2: Basic Definitions and Algebraic Geometry

In this talk the notion of a large field given in the introduction of [5] should be defined. For that first give a brief reminder of the notion of smooth points on varieties. This should preferably be done in the classical setting (cf. [6, Chapter 2]). Then easy examples of large fields should be given like $\mathbb{C}$ and $\mathbb{R}$ and it should be explained why they are large. Also at least one non-example should be given - for instance $\mathbb{Q}$. This is especially interesting as it explains one of the many names for large fields, namely *anti-mordellic* as one general counterexample could be a curve of genus $g > 1$ by Faltings famous theorem (cf. [1]) or more specifically consider the variety given by $X^2 = Y^2 + 1$, then by the long known instance for $n = 4$ of Fermats last theorem, this only has finitely many rational points.

Talk 3: Characterizations of largeness

This talk should give a brief introduction to the model-theoretic notion of existentially closedness, then explain different equivalent characterizations of large fields, especially [5, Proposition 2.1.] and [5, Proposition 2.4] (omitting the very model-theoretic proof) and [5, Proposition 2.6]. It should be explained why algebraic extensions of large fields are large [5, Proposition 2.7].

Talk 4: Examples

In this talk the definition of a henselian valued field should be given (cf. [2]), the algebraic implicit function theorem explained and deduced that henselian valued fields are large. Also the notion of a pseudo algebraically closed fields should be introduced and the proof should be given that they are large.

[1] Coincidentally also one of the many names for large fields

Talk 5 & 6: Inverse Galois Problem

These talks should follow [5, Part II Section 4] as well as [4] and explain the inverse Galois problem and the proof of Main Theorem A in [4]. They should be prepared in close coordination.

References

[1] Bombieri, E. The Mordell conjecture revisited. http://www.numdam.org/item/ASNSP_1990_4_17_4_615_0.pdf. 1

[2] Engler, A. J., and Prestel, A. *Valued Fields*. Springer Monographs in Mathematics. Springer. 1

[3] Johnson, W., Walsberg, E., and Ye, J. The étale open topology over the fraction field of a henselian local domain. 1

[4] Pop, F. Embedding Problems Over Large Fields. *Annals of Mathematics 144*, 1, 1–34. 1, 2

[5] Pop, F. Little survey on large fields – Old & New –. https://www2.math.upenn.edu/~pop/Research/files-Res/LF_6Oct2013.pdf. 1, 2

[6] Shafarevich, I. R. *Basic Algebraic Geometry 1*. Springer Berlin Heidelberg. 1