Problems for Tutorial 2

(Thursday, 25.11, at 10 a.m.)

Problem 1.

a) Prove that the action of $\text{Möb}_{\mathbb{R}}$ on $\mathbb{H}$ is 1-transitive[1]. [3 P.]

b) Prove that the action of $\text{Möb}_{\mathbb{R}}$ on $\mathbb{H}$ is not 2-transitive. [2 P.]

c) Compute the stabilizer of i in $\text{Möb}_{\mathbb{R}}$: [3 P.]

$$\text{St}_{\text{Möb}_{\mathbb{R}}}(i) := \{T \in \text{Möb}_{\mathbb{R}} \,|\, T(i) = i\}.$$

d) Let r be an arbitrary positive real number. Prove that $\text{St}_{\text{Möb}_{\mathbb{R}}}(i)$ acts 1-transitive on the circle $\{z \in \mathbb{H} \,|\, \rho(z,i) = r\}$. [8 P.]

Problem 2. For the map $\eta : \mathbb{H} \to \mathbb{H}$, $z \mapsto -\bar{z}$, prove the folowing statements.

a) $\eta \in \text{Isom}\mathbb{H}$. [2 P.]

b) $\eta \notin (\text{Möb}_{\mathbb{R}})_{|\mathbb{H}}$. [2 P.]

Problem 3. Prove Lemma 1.4.2 from the script:
Isometries of $\mathbb{H}$ map geodesic lines (i.e. lines of the form $\mathbf{A}_r$ and $\mathbf{C}_{r_1,r_2}$) to geodesic lines. [6 P.]

Problem 4. For two different points $u, v \in \mathbb{H}$, the set

$$\text{Equidist}(u,v) := \{z \in \mathbb{H} \,|\, \rho(z,u) = \rho(z,v)\}$$

is called *Equidistance* of u and v.

a) Prove that the Equidistante coincides with a geodesic line of the form $\mathbf{A}_r$ or $\mathbf{C}_{r_1,r_2}$. [4 P.]

b) Give an exact formula for $\text{Equidist}(1+i, 3+3i)$. [3 P.]

c) Draw the set from b). [1 P.]

Hint. Use the formula (2) or (3) from Theorem 1.3.8 of the script.

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[1] Let G be a group acting on a set X. This action is called n-transitive if for every two tuples $(x_1, \ldots, x_n)$ and $(x'_1, \ldots, x'_n)$ of elements from X satisfying $x_i \neq x_j$ and $x'_i \neq x'_j$ for all $i \neq j$, there exists an element $g \in G$ such that $g(x_1) = x'_1, \ldots, g(x_n) = x'_n$.